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# *MATHEMATICAL MODEL OF PARTICLE ENTRAINMENT FROM A FLUIDIZED BED*

*The paper proposes a one-dimensional mathematical model for describing the movement of liquefying air and the movement of particles in the gas-solids fluidized bed apparatus. The modeling is based on the mathematical approach of the theory of counting Markov chains. The factor determining the material entrainment from the apparatus was introduced into the model by using a macro diffusion coefficient, which made it possible to supplement the matrix of transition probabilities with probabilities of random throwing of particles into neighboring cells of the chain. Numerical experiments performed with the developed model allowed us to obtain qualitatively consistent results with respect to the distribution of the solid phase and the gradual removal of particles from the apparatus. The results obtained allow to consider the proposed model as a reliable basis for the development of a computer method for calculating the expansion of the bed and the entrainment of particles from technological equipment.*
*Keywords: fluidized bed, Markov chain theory, macrodiffusion, entrainment, mathematical modeling.*

**Introduction**

The principle of fluidization of a particulate medium by a gasifying agent and the technique using this principle penetrated into the energy industry when some experience had already been accumulated in the chemical and related industries. In part, this experience was associated with the accumulation of some claims to classical fluidization schemes. With this in mind, the vector of development of fluidization technology in the energy sector was initially directed towards forcing hydrodynamic regimes [1-3]. However, the search for more intensive hydrodynamic fluidization regimes is inextricably linked with an increase in the entrainment of material from the equipment, which to a certain extent is limited by the organization of the looping of the phases of the bed [3–4]. Nevertheless, devices with a traditional fluidized bed have occupied their niche in the energy sector and are used in fuel treatment and combustion systems [3–5]. In this regard, the task of constructing and investigating models of particle entrainment from the apparatus remains urgent.

Various mathematical approaches are used to simulate the operation of fluidized bed apparatuses [4–8], among which cell models occupy an increasingly expanding niche [9–10]. However, these models were developed either for the study of conditionally periodic fluidization (without entrainment of particles) [9] or a circulating fluidized bed [10]. Thus, the possibility of modeling the operation of the periodic principle of operation apparatus (in which undesirable entrainment of particles is almost always observed) has not been investigated.

**Materials and methods**

The mathematical apparatus of the Markov chain theory is used here as a computational basis for constructing the model. The main provisions of the model were proposed earlier [9-10], so here they will be set out only in the main provisions. The space of the apparatus is represented by a set of a countable number of n cells organized into a chain, which is characterized by a vector of states of the system S. The evolution of state vectors is calculated for discrete time points $t_k=(k-1)\Delta t$, where $\Delta t$ is the time step, k is the time step number.

$$S_p^{k+1}=P_p^{\ k}\, S_p^{\ k}, \tag{1}$$

where $P_p$ is the matrix of transition probabilities.

The matrix $P_p$ is a tri-diagonal matrix, elements of which are calculated as: probability of downward transitions for the particles in i-th cell

$$P_{p,i-1,i}=d+v_i^k\cdot(1-e), \quad i=\overline{1, n-1}, \tag{2}$$

probability of upward transitions for the particles in i-th cell

$$P^k_{p,i+1,i}=d+v_i^k\cdot e, \quad i=\overline{1, n-1}, \tag{3}$$

probability to stay for the particles in i-th cell

$$P^k_{p,i,i}=1- P^k_{p,i+1,i} - P_{p,i,i+1}, \quad i=\overline{1, n}, \tag{4}$$

where *d* is the probability of pure stochastic (diffusion) transitions, $v_i^k$ is the probability of convection transition caused by the gas upstream flow, if $v_i^k>0$ e=1 and if $v_i^k<0$ e=0.

The probability of stochastic transitions can be defined as:

$$d=D\Delta t/\Delta x^2, \tag{5}$$

where D is the dispersion coefficient.

The asymmetrical transition probability can be defined as:

$$v_i^k=(W_i^k - V_{sj}^{\ k})\, \Delta t/\Delta x^2, \tag{6}$$

The state of the gas phase is also described as distributed over a chain of cells. The state vector for the gas Sg changes according to a similar recurrent procedure

$$S_g^{k+1}=P_g^{\,k}\,S_g^{\,k}+S_{gf},\qquad (7)$$

where $P_g$ is the matrix of transition probabilities for gas, $S_{gf}$ is the vector of gas source.

Assuming that the gas flow moves only upwards, it is possible to write down the probabilities of transitions along the chain, organizing them into a transition matrix Pg, in the following form

probability of upward transitions

$$P^k_{g,i+1,i}=v_i^k,\ \text{for } 1\le i<n-1,\qquad (8)$$

probability to stay

$$P^k_{g,i,i}=1-v_i^k,\ \text{for } 1\le i<n,\qquad (9)$$

where $v_i^k=W_i^k\Delta t/\Delta x$ is the asymmetrical part of probability transition along the chain for gas, $W_i$ is local gas velocity in the i-th cell.

The principles of identification of model parameters used further are described in detail in [9–10].

**Results and discussion**

Figure 1 shows a computational scheme for modeling the motion of phases in a fluidized bed.

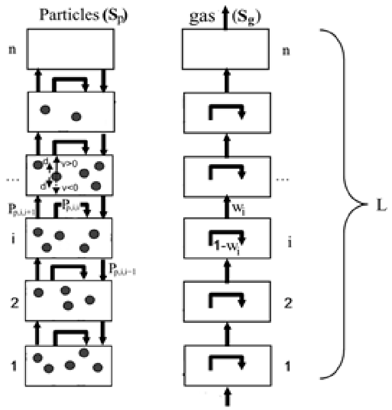


Figure 1 – Calculation scheme of a the mathematical model

In the simulation, the device was represented by chains of 10 cells with a height of Δx=0,1 m. The amount of material in the last cell of the chain was stored in a separate array (this is the entrainment vector). As the particles are removed from the apparatus (at the initial moment of time there was 3 kg in it), the state of the fluidized bed changes, while asymptotically the weight of the particulate solid in the apparatus tends to zero. In this regard, the study of the transition process is of greater interest.

Figure 2 shows the distribution of particles along the bed height for a time of 15 minutes, as well as the change in the intensity of particle entrainment during 30 minutes of fluidization.

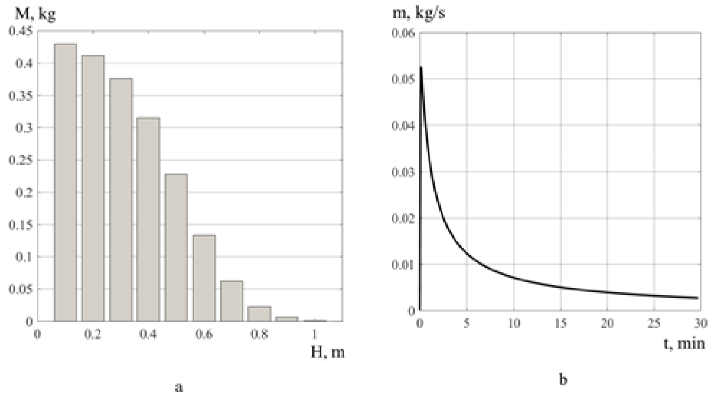


Figure 2 – distribution of the solid phase along the height of the fluidized bed (a); intensity of particle entrainment from the apparatus (b)

As can be seen, the distribution of particles along the height of the bed, due to the presence of stochastic probabilities of particle motion, has a completely trivial appearance (Fig. 2a), while the particle entrainment rate has a maximum with complete liquefaction of the initial loading mass, and then decreases, tending to zero (Fig. 2b).

**Conclusions**

The study proposes a cellular mathematical model of the fluidized bed state, in which the introduction of a random probability of transfer between cells makes it possible to obtain a qualitatively consistent distribution of the phases of the bed over the height of the layer and at the calculated level to explain the mechanism of entrainment of solid phase particles from the apparatus.

## References

1 **Мухленов, И. П.** Расчеты аппаратов кипящего слоя : справочник / И. П. Мухленов, Б. С. Сажин, В. Ф. Фролов; под ред. И. П. Мухленова. – Л. : Химия, 1986. – 352 с.

2 **Митрофанов, А. В., Шуина, Е. А., Камело, А. Ф., Шумыло, Е. Р.** Разработка вероятностно-статистической модели сушки частиц биотоплива в кипящем слое инертного носителя // Вестник Ивановского государственного энергетического университета. – 2018. – № 3. – С. 57–64.

3 **Oka, S. N.** Fluidized Bed Combustion. – NY : Basel, Marcel Dekker, 2004. – 599 p.

4 **Lu, X., Li, S., Du, L., Yao, J., Lin, W., Li, H.** Flow structures in the downer circulating fluidized bed // Chemical Engineering Journal. – 2005. – Vol. 112. – Issues 1–3. – P. 23–31.

5 **Wen-Chin, Y.** Handbook of fluidization and fluid-particle systems / NY : MarcelDekker Inc., 2003. – 878 p.

6 **Баскаков, А. П., Мунц, В. А.** Сжигание твердых топлив и горючих отходов в котлах со стационарным и циркуляционным кипящим слоем // Химическая промышленность. – 1993. – № 12. – С. 61-65.

7 **Zhonghua, W., Mujumdar, A. S.** CFD modeling of the gas-particle flow behavior in spouted beds // Powder Technology. – 2008. – Vol. 183. – P. 260–272.

8 **Deen, N. G., van SintAnnaland, M., van der Hoef, M. A., Kuipers, J. A. M.** Review of discrete particle modeling of fluidized beds // Chem. Eng. Sci. – 2007. – Vol. 62. – P. 28–44.

9 **Mitrofanov, A. V., Mizonov, V. E., Tannous, K., Ovchinnikov, L. N.** A Markov chain model to describe fluidization of particles with time-varying properties // Particulate Science and Technology. – 2018. – Vol. 36. – No. 2. – P. 244–253.

10 **Mitrofanov, A. Mizonov, V., Camelo, A., Tannous, K.** Application of the theory of Markov chains to theoretical study of processes in a circulating fluidized bed // Particulate Science and Technology. – 2019. – Vol. 37. – No. 8. – P. 1028–1033.

## References

1 **Muhlenov, I. P.** Raschety apparatov kipyashego sloya: Spravochnik [Calculation of fluidized bed reactors. Hand-book] / I. P. Muhlenov, B. S. Sazhin, V. F. Frolov; pod red. I. P. Muhlenova. – L. : Khimiya, 1986. – 352 p. (in Russian)

2 **Mitrofanov, A. V., Shuina, E. A., Camelo, A. F., Shumylo, E. R.** Razrabotka veroyatnostno-statisticheskoj modeli sushki chastic biotopliva v kipyashchem sloe inertnogo nositelya [Stochastic modeling of biofuel particulate drying in the fluidized bed with inert support] // Vestnik IGEU. – 2018. – № 3. – P. 57–64. (in Russian)

3 **Oka, S. N.** Fluidized Bed Combustion. – NY : Basel, Marcel Dekker, 2004. – 599 p.

4 **Lu, X., Li, S., Du, L., Yao, J., Lin, W., Li, H.** Flow structures in the downer circulating fluidized bed // Chemical Engineering Journal. – 2005. – Vol. 112. – Issues 1–3. – P. 23–31.

5 **Wen-Chin, Y.** Handbook of fluidization and fluid-particle systems / NY : MarcelDekker Inc., 2003. – 878 p.

6 **Baskakov, A. P., Munts, V. A.** Szhiganie tverdyh topliv i goryuchih othodov v kotlah so stacionarnym i cirkulyacionnym kipyashchim sloem [Combustion of solid fuels and combustible waste in boilers with stationary and circulating fluidized bed] // Himicheskaya promyshlennost'. – 1993. – № 12. – P. 61–65.

7 **Zhonghua, W., Mujumdar, A. S.** CFD modeling of the gas-particle flow behavior in spouted beds // Powder Technology. – 2008. – Vol. 183. – P. 260–272.

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## ҚАЙНАҒАН ҚАБАТЫ БАР АППАРАТТАН БӨЛШЕКТЕРДІ ТАСЫМАЛДАУДЫҢ МАТЕМАТИКАЛЫҚ МОДЕЛІ

*Жұмыста сұйытылған ауаның қозғалысын және қайнаған қабат аппаратындағы бөлшектердің қозғалысын сипаттайтын бір өлшемді математикалық модель ұсынылған. Модельдеудің негізі Марковтың санау тізбегі теориясының математикалық аппараты болып табылады. Құрылғыдан материалдың тасымалдануын анықтайтын Фактор модельге макро диффузиялық коэффициентті қолдану арқылы енгізілді, бұл өтпелі ықтималдық матрицасын іргелес тізбек* *ұяшықтарына бөлшектердің кездейсоқ құйылу ықтималдығымен толықтыруға мүмкіндік берді. Әзірленген модельмен жүргізілген сандық эксперименттер қатты фазаның таралуына және бөлшектердің аппараттан біртіндеп шығарылуына қатысты сапалы дәйекті нәтижелерге қол жеткізді. Алынған нәтижелер ұсынылған модельді қабаттың кеңеюін және бөлшектердің технологиялық аппаратурадан шығуын есептеудің компьютерлік әдісін дамытудың сенімді негізі ретінде қарастыруға мүмкіндік береді.*

*Кілтті сөздер: қайнаған қабат, Марков тізбектерінің теориясы, макродиффузия, алып кету, математикалық модельдеу.*

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# МАТЕМАТИЧЕСКАЯ МОДЕЛЬ УНОСА ЧАСТИЦ ИЗ АППАРАТА С КИПЯЩИМ СЛОЕМ

*В работе предложена одномерная математическая модель для описания движения ожижающего воздуха и перемещения частиц в аппарате кипящего слоя. Основой моделирования служит математический аппарат теории счетных цепей Маркова. Фактор, определяющий унос материала из аппарата, введен в модель за счет использования макродиффузионного коэффициента, который позволил дополнить матрицу переходных вероятностей вероятностями случайного заброса частиц в соседние ячейки цепи. Выполненные численные эксперименты с разработанной моделью позволили получить качественно непротиворечивые результаты в отношении распределения твердой фазы и постепенного выноса частиц из аппарата. Полученные результаты позволяют рассматривать предложенную модель в качестве достоверной основы для развития компьютерного метода расчета расширения слоя и уноса частиц из технологической аппаратуры.*

*Ключевые слова: кипящий слой, теория цепей Маркова, макродиффузия, унос, математическое моделирование.*

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