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Features of Application of Simulation Modeling of the Process of Forming the Accuracy of Required Dimensions

A. I. Denchik[a], *, A. Zh. Kassenov[a], **, A. S. Yanyushkin[b], *, Zh. K. Musina[a], ****, K. K. Abishev[a], *****, D. A. Iskakova[a], ******, and A. A. Tkachuk[a], *******

[a] *Toraighyrov University, Pavlodar, 140000 Kazakhstan*
[b] *I.N. Ulianov Chuvash State University, Cheboksary, 428003 Russia*
**e-mail: pavlodarec99@mail.ru*
***e-mail: asylbek_kasenov@mail.ru*
****e-mail: yanyushkinas@mail.ru*
*****e-mail: mussina_zhanara@mail.ru*
******e-mail: a.kairatolla@mail.ru*
*******e-mail: iskakovada@mail.ru*
********e-mail: tkachukandrey1985@mail.ru*

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Abstract—The numerous factors that influence the technological process and cause errors during manufacture of parts complicate the problem of ensuring the required accuracy of required dimensions. The article proposes a method for determining coefficients of simulation models using probabilistic statistical methods, the versatility of which makes them applicable to a wide range of problems.

Keywords: accuracy, scale factor, probabilistic model, simulation modeling, technological disturbance
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Ensuring the required accuracy during manufacture of parts is complicated by the large volume of statistical data used as the scientific and applied basis for doctrines for the formation of geometric accuracy indicators [1–9]. The task of ensuring accuracy is made highly complex by the need to adjust for a huge number of factors that influence the technological process and a wide range of properties of required dimensions. Therefore, for these purposes it is advisable to rely on probabilistic statistical methods that result in the development of a probabilistic model [5].

The results of simulation modeling (SM) [5] have been used to formulate an indicator called the average coefficient $C_{\text{avg.SM}}$ of relative technological disturbances, which describes compliance of the conditions for the formation of an array of possible values of required dimensions L_i of m elements under the technical and technological features of the performed operation, i.e., coefficient $C_{\text{avg.SM}}$ describes the intensity of disturbances in the actual shaping process caused by effects of random factors.

It is assumed that coefficient $C_{\text{avg.SM}}$ is a conditional unit of accuracy of the shaping process. The unit of accuracy $\Delta\delta_i = C_{\text{avg.SM}}$ is the average number of disturbances per unit length of required dimensions L_i per test for an array of possible values of required dimensions L_i of m elements formed under constant SM conditions and adjusted for the technical and technological features of the operation.

The average number of disturbances per unit of required dimensions L_i for the entire cycle of forming its accuracy (i.e., for the total number of tests n_{ttl}) for the considered technological operation is:

$$C_{\text{SM}} = n_{\text{ttl}} C_{\text{avg.SM}}. \quad (1)$$

Then the accuracy of required dimensions in SM is determined by the formula

$$\delta_{L_i} = P_{L_i\text{SM}} C_{\text{SM}}, \quad (2)$$

where $P_{L_i\text{SM}}$ is the probability of disturbances of required dimensions.

The values of probability $P_{L_i\text{SM}}$ in SM and probability P_{L_i} obtained by analytical analysis [6, 10], as well as the values of coefficient C_{SM} in SM and coefficient C obtained by analytical analysis [10], have the same structure and the same physical significance; therefore, the final expression for model (2) has the form:

$$\delta_{L_i} = \left[1 - e^{-CL}\right] CK, \quad (3)$$

where K is an indicator of similarity of the accuracy of required dimensions L_i in SM and during mechanical processing (MP).

Analysis of model (3) showed that error δ_{L_i} of required dimensions is proportional to coefficient C of relative technological disturbance for this operation and probability P_{L_i} of disturbances in the formation of the accuracy of required dimensions after adjustment for the scale factor.

Before considering variations of the specific problem in terms of the methodology for determining indicators C and K, let us first analyze the features of determining similarity indicator K.

The numerical value of K is determined based on the basic principles of dimensional analysis, according to which:

(1) the dimensions of the parameter of the object of analysis (OA) and the dimensions of the model must be the same, unless the compared values are dimensionless;

(2) the numerical values of the experimental OA and the OA calculated according to model (3) must be the same at a similar point (or points).

Then the similarity conditions for solving this problem have the form:

$$\delta = \left(1 - e^{-CL}\right)C; \quad \delta \mathrm{P_o} = \left(1 - e^{-CL}\right)C\mathrm{P_M};$$
$$\delta[L]\mathrm{P_o}[1/L] = \left(1 - e^{-CL}\right)C[1/L]\mathrm{P_M}[L];$$
$$\mathrm{P_M}/\mathrm{P_o} = K\left[L^2\right]; \quad \delta[L] = \left(1 - e^{-CL}\right)C[1/L]K\left[L^2\right],$$

where $\mathrm{P_M}$ is the dimensional similarity parameter of the model; $\mathrm{P_o}$ is the dimensional similarity parameter of the OA; and δ is the error calculated using the model.

The sequence of determining the values of indicators C and K depends on whether accuracies δ at similar (reference) points of the OA and the model are known and on the number of such points. Therefore, based on the above, let us analyze the order and features of deriving the numerical values of C and K for the most common variations of practical problems.

Let us consider a version of determining the numerical values of C and K when there are no a priori values of processing accuracy at similar points of the OA and the SM (3).

The absence of accuracy values at similar points of the OA and the SM suggests the following. Consider the construction of an informative dependence $\delta = f(C, L, K)$ in the range of expected variation of required dimensions L, which represents the influence of indicators K and L on accuracy of δ at $C = \text{const}$. In this case, at the first stage, for a random L, it is necessary to determine the variation interval $0 \le L \le L_{\mathrm{pi}}$, where L_{pi} is the maximum required dimensions defining the boundary of the range, chosen based on production interests.

Since there are no similar points of the OA and the SM, the value of the similarity index is for convenience assumed to be $K = 1$.

Assign a confidence level for the values of required dimensions L being within the interval $0 \le L \le L_{\mathrm{pi}}$.

The confidence level of required dimensions is selected based on the condition that a random variable occurs in the considered range. For example, according to the three-sigma rule, at a probability of 0.9973, the occurrence of all expected values in the considered range is 99.73%.

Having determined the interval of variations of random variable L, confidence level $P(0{-}L_{\mathrm{pi}}) = 0.9973$, and coefficient $K = 1$, let us derive indicator C for model (3).

According to [4], the probability of an exponential continuous random variable X occurring in interval (a, b) is defined by the distribution function

$$F(X) = 1 - e^{-(\lambda x)} \text{ at } x > 0$$

and is determined as

$$P(a < X < b) = F(b) - F(a).$$

Given that $F(a) = 1 - e^{-\lambda a}$ and $F(b) = 1 - e^{-\lambda b}$:

$$P(a < X < b) = e^{-\lambda a} - e^{-\lambda b}.$$

At $a = 0$:

$$P(0 < X < b) = e^{-\lambda 0} - e^{-\lambda b} = 1 - e^{-\lambda b}.$$

Then the expression for determining the probability of continuous random variable L occurring in the range of possible values $0 \le L \le L_{\mathrm{pi}}$ takes the form:

$$P(0 < L < L_{\mathrm{pi}}) = 1 - e^{-CL_{\mathrm{pi}}}.$$

At the accepted $P_{L_i} = 0.9973$:

$$0.9973 = \left[1 - e^{-CL_{\mathrm{pi}}}\right]$$

or

$$0.0027 = e^{-CL_{\mathrm{pi}}}; \quad C = \frac{\ln\left(\frac{1}{0.0027}\right)}{L_{\mathrm{pi}}}; \quad C = \frac{\ln(370.370)}{L_{\mathrm{pi}}}. \tag{4}$$

The sequence of deriving indicator C in the absence of a value of δ at a similar point:

(1) determine the interval for investigating the random variable: $0 \le L \le L_{\mathrm{pi}}$, where L_{pi} is the maximum possible required dimensions under the relevant production conditions, corresponding to the right boundary of the analysis range;

(2) select the confidence level $P(0-L_{pi})$ for the considered range of required dimensions $0 \le L \le L_{pi}$.

(3) calculate C using formula (4).

The advantage of this approach to deriving indicator C is that it enables constructing a family of informative dependencies $\delta = f(C, L, K)$ in the range $0 \le L \le L_{pi}$ for $L_{pi} > 0$, which, by varying K in the range $0 < K < \infty$ and given an available accuracy value at similar points of the SM and the OA, can be adapted to any result of the study of the OA. In addition, the dependence $\delta = f(C, L, K)$ makes it possible to use δ as the numerical value of the closing link of the dimensional chain machine-device-tool-workpiece (MDTW) in equipment design.

With known accuracy at several similar points of the OA and the SM, the final numerical value of K is selected based on the results of correlation analysis from the condition of its maximum value or from the condition of the minimum sum of squares of deviations of accuracy at similar OA-SM points according to the least squares method.

Let us consider determination of the numerical value of indicator C at known accuracy δ. There are two possible options:

(1) the value of δ_e for the OA is known at one similar point of the OA ($n = 1$);

(2) more than one value of δ_e for the OA, at several similar points of the OA, are known ($n > 1$).

For a known value of δ_e, the solution to the problem is reduced to solving the equation

$$\delta_e = \left[1 - e^{-CL}\right]CK,$$

and for several known values, a system of equations:

$$\delta_{e1} = \ldots;$$
$$\delta_{e2} = \ldots;$$
$$.$$
$$.$$
$$.$$
$$\delta_{en} = \ldots.$$

The difference is that for a set of known values δ_{ei}, the final value of C is determined using the least squares method, i.e., the sum of the squares of the difference between δ_{ei} and δ_{ci} tends to the minimum:

$$\sum_{i=1}^{n}\left(\delta_{ei} - \delta_{ci}\right)^2 \to \min.$$

There are several possible methods to determine C in equation (3) for known values of δ_e:

(1) trial and error;

(2) analytically, by solving the equation and finding an expression to determine the root of the original equation C;

(3) graphically, by plotting the experimental (δ_e) and calculated (δ_c) values and determining the point of their intersection, which is the root of the original equation.

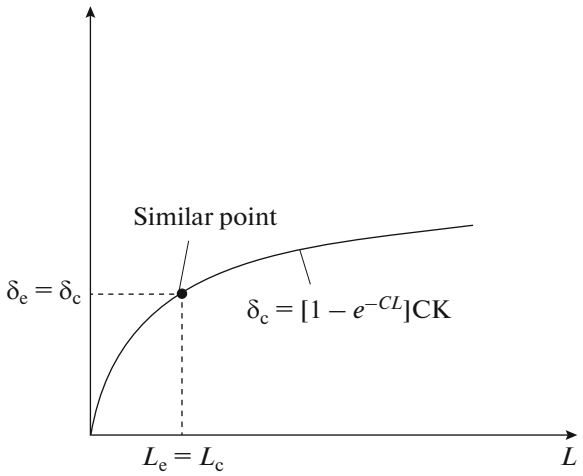

Fig. 1. Graphical solution of the original equation.

These methods of determining C are described in detail in [11, 12]. Let us focus on only the specific features of their application to the considered problem.

For the number of similar points $n = 1$ and 2, there are two approaches to determining C and K to fulfill the condition of similarity between the SM and the OA $\delta_{ei} = \delta_{ci}$: the first approach is assignment of K with subsequent correction of C, the second is determination of C with subsequent correction of K. Additionally, due to the specific features of the study, the possibility of joint combined use of these methods is not excluded.

The choice of determination of C or K or their combined use depends on features of the study, and the final decision is made by the researcher.

Let us analyze the application of these methods for one similar point, which implies an unambiguous solution of the original equation (3) from the condition $\delta_e = \delta_c$ at one similar point at $L_e = L_c$ and $K = 1$ (Fig. 1).

Having determined C at $K = 1$ by expression (3) at similar points, we can now calculate and predict accuracy values δ_i for the entire range of required dimensions in the range $0 \le L \le L_{pi}$.

Let us derive C and K at known accuracy values δ_{e1} and δ_{e2} at two similar OA-SM points for required dimensions L_1 and L_2 respectively.

Derivation of C and K by formula (3) is done by optimizing objective functions (OF) via a series of successive iterations of δ_{e1} and δ_{c1}, δ_{e2} and δ_{c2} while varying C and K.

The first sequence of determining C and K at two known experimental accuracy values δ_{e1} and δ_{e2} for the

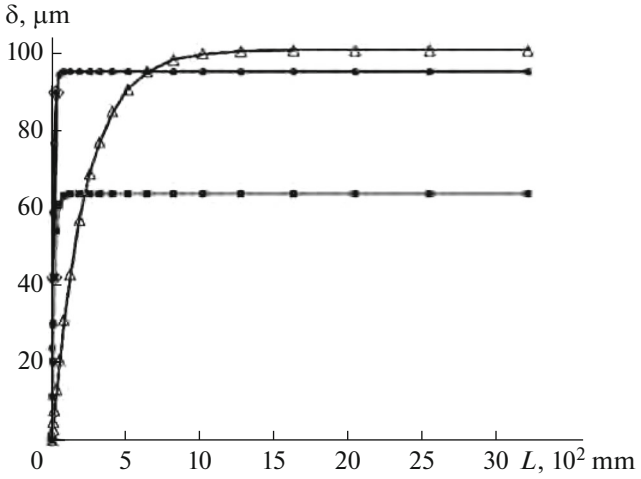

Fig. 2. Dependencies of changes in accuracy parameters δ_e (–◇–), δ_{acc} (–Δ–), δ_{init1} (–●–), δ_{init2} (–■–) on required dimensions L.

corresponding values L_1 and L_2 of required dimensions:

(1) preliminary determination of the C_{acc} from the condition $P(0-L_{pi}) = 0.9973$ at $K = 1$;

(2) determination of the numerical value of C_1 from the condition $OF = \delta_{e1} - \delta_{c1} = 0$ at $K = 1$;

(3) determination of the numerical value of C_2 from the condition $OF = \delta_{e1} - \delta_{c1} = 0$ at $K = 1$;

(4) adjustment of the numerical value of K from the condition of the minimum value of the OF of the sum of squares of the difference:

$$OF = (\delta_{e1} - \delta_c)^2 \rightarrow \min$$

or

$$OF = (\delta_{e2} - \delta_c)^2 \rightarrow \min$$

according to the least squares method for a corresponding value of $C = C_1$ or $C = C_2$ by varying parameter K.

The second sequence:

(1) preliminary determination of $C = C_{acc}$ from the condition $P(0-L_{pi}) = 0.9973$ at $K = 1$;

(2) determination of the numerical value of K from the condition $OF = \delta_e - \delta_c = 0$ for points *1* and *2*;

(3) adjustment of indicator C by minimizing $OF = (\delta_e - \delta_c)^2$ for points *1* and *2*;

(4) adjustment of indicator K by minimizing $OF = (\delta_e - \delta_c)^2$ for points *1* and *2*;

The choice of the sequence for determining C and K depends on features of the specific study, and the final decision is made by the researcher.

Let us consider the option of determining the numerical values of C and K at known accuracy values when there are more than two similar points ($n > 2$). In this case, derivation of C is based on analysis of the correlation coefficient between the set of accuracy values of the OA (δ_{ei}) and calculation of the accuracy values of the SM δ_{ci}, followed by adjustment of the value of K until the numerical equality $\delta_{ei} = \delta_{ci}$ is achieved.

Based on the above, the sequence of derivation is as follows:

(1) preliminary determination of C_{acc} from the condition $P(0-L_{pi}) = 0.9973$ at $K = 1$;

(2) preliminary determination of the correlation coefficient KK_{acc} between experimental (a priori) and calculated values of tolerance accuracy δ_e and δ_c;

(3) adjustment of C_r from the condition of optimization of the values of the objective function $OF = KK_{acc}$ by varying C up to the value of $KK_{acc} = KK_{max} = 0.97$;

(4) adjustment of the numerical value of K_r from the condition of the minimum value of the sum of squares of the difference $(\delta_e - \delta_c)^2$ using the least squares method.

Sometimes the numerical values of C and K in model (3), determined based on the known accuracy values δ of the SM and the OA for one, two, or many similar points, render graphical dependencies $\delta = f(C, L, K)$ inconvenient in terms of clearly conveying information. Therefore, in such cases the numerical value of C can be adjusted to an acceptable value $(C \rightarrow C_{acc})$ while maintaining an acceptable level of correlation KK_b between the initial accuracy values δ_{init} of required dimensions L obtained before correction and acceptable accuracy values δ_{acc} obtained after correction. Assume an acceptable level of correlation (or the minimum possible correlation coefficient), for example, $KK_b = 0.75$.

Correction of C is valid because, firstly, the exponential nature of the dependence $\delta = f(C, L, K)$ remains, and numerical changes of δ_{acc} are in an acceptable range, and secondly, the value of the correlation coefficient remains within acceptable limits.

The numerical value of coefficient K according to the least squares method is determined from the condition of the minimum value of the objective function: $OF = (\delta_{init} - \delta_{acc})^2 \rightarrow \min$.

As an example, consider correction of two processes with the parameters $C_1 = 0.095447$ and $K_1 = 1$; $C_2 = 0.063573$ and $K_2 = 1$ and with the accepted values C_{acc} and K_{acc} at $\delta_{e1} = 42$ μm, $\delta_{e2} = 90$ μm, $L_1 = 17$ mm, $L_2 = 25$ mm.

The dependences of changes in accuracy parameters δ_e, δ_{acc}, δ_{init1}, δ_{init2} on required dimensions L are shown in Fig. 2. The numerical values of the parameters of the corrected process $C_{acc} = 0.004587$ and $K_{acc} = 22.00$, while the correlation coefficients of the

corrected process with the initial processes were $KK_{acc1} = 0.689$; $KK_{acc2} = 0.753$.

Thus, it has been shown that the procedure of determining the numerical values of C and K depends on availability of known accuracy values δ at similar OA-SM points and on the number of these points. The proposed derivation of the values of C and K for the SM is applicable to the most common variants of the problem.

In the absence of accuracy values at similar points of the OA and the SM, the procedure is to determine the interval of variation of the random variable $0 \le L \le L_{pi}$, select the confidence level $P(0-L_{pi})$ for the considered range of required dimensions, calculate C using formula (4), and construct the dependence $\delta = f(C, L, K)$.

For $n = 1$ and 2, we consider two approaches to determining C and K that take into account the similarity condition for the SM and the OA $\delta_{ei} = \delta_{ci}$: assignment of K with correction of C, determination of C with correction of K. For $n > 2$, determination of C is based on analysis of the correlation coefficient between the set of accuracy values δ_{ei} of the OA and the calculated values δ_{ci} with adjustment of coefficient K to achieve the equality $\delta_{ei} = \delta_{ci}$.

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CONFLICT OF INTEREST

The authors of this work declare that they have no conflicts of interest.

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