

Simulation in Establishing the Dimensional Precision of Machine Parts

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Abstract—Approaches to guaranteeing the dimensional precision of machine parts are considered. It is difficult to formulate a uniform approach on account of the limited statistical data on which the corresponding theory may be based. In addition, numerous factors determining the operational errors in the manufacturing process must be taken into account. Accordingly, it is expedient to employ universal statistical and probabilistic methods in addressing the reliable dimensional precision of products.

Keywords: dimensional precision, machine parts, scale factor, probabilistic model, simulation, technological disturbances, similarity coefficient, manufacturing

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It is difficult to formulate a uniform approach in dimensional analysis of prospective products on account of the potential range and variability of their characteristics [1–3]. In guaranteeing the manufacturing precision of machine parts, another complicating factor is that the statistical data on which the corresponding theory may be based are limited [4–8]. In addition, numerous factors determining the operational errors in the manufacturing process must be taken into account. Accordingly, it is expedient to employ universal statistical and probabilistic methods in addressing the reliable dimensional precision of products [9–12].

In the present work, we present examples in which the coefficient C characterizing the influence of random factors on a specific technological operation and the scale factor K of the actual accuracy and the model accuracy with known accuracy at similar points for the most common models and objects of analysis.

On the basis of simulation, the concept of the mean relative technological perturbation $C_{me,si}$ may be formulated [4]. In our view, $C_{me,si}$ characterizes the similarity of the conditions in which the set of possible dimensional values is formed (m elements), with given

characteristics of the shaping operation, and describes the degree of perturbation of the actual shaping process on account of random factors.

We assume that $C_{me,si}$ is the unit of accuracy $\Delta\delta_i$ of the shaping process: $\Delta\delta_i = C_{me,si}$. In physical terms, $C_{me,si}$ is the mean number of perturbations per unit dimensional length in one test for the set of possible dimensional values (m elements) corresponding to fixed modeling conditions or characteristics of the shaping operation.

The coefficient C_{si} characterizes the perturbation over the whole operational cycle

$$C_{si} = n_{tot} C_{me,si}. \quad (1)$$

In physical terms, C_{si} is regarded as the mean number of perturbations per unit dimensional length over the whole cycle n_{tot} in which its accuracy is formed within the given operation, for the adopted unit of shaping accuracy $\Delta\delta_i = C_{me,si}$. In that case, taking account of Eq. (1), the dimensional precision in simulation is determined from the following formula, according to [9]

$$\delta_{L_i} = P_{L_i,si} C_{si} K, \quad (2)$$

where K is the similarity of the precision of dimension L_i in simulation and in machining.

The physical meaning of the probabilities $P_{L_i,si}$ and P_{L_i} of dimensional perturbations in simulation and analytical calculation, respectively, and the coefficients C_{si} and C is the same [9]. Therefore, the final form of Eq. (2) is

$$\delta_{L_i} = [1 - e^{-(CL)}]CK. \tag{3}$$

Analysis of the model in Eq. (3) indicates that the dimensional error δ_{L_i} is proportional to the relative technological perturbation C for the given operation and the probability $P_{L_i} = 1 - \exp(-CL)$ of perturbation in establishing the dimensional accuracy, taking account of the scale factor.

Theoretically, in solving practical and applied problems, the coefficient C may be determined when $K = 1$ if we know the precision for just one similar point: $n = 1$. However, to improve the reliability of the dependence $\delta = f(C, K, L)$ and the results obtained, we consider examples in which C and K are determined with two or more similar points: $n = 2$ and $n > 2$.

With a single similar point, C and K may be determined by two methods: 1) by variation of C ; 2) by variation of K .

The first method includes the following steps.

1. Preliminary determination of C_{pr} from the condition that $P(0, \dots, L_{pi}) = 0.9973$

$$C_{pr} = \ln 370.370/L_{pi} \quad \text{when } K = 1;$$

$$C_{pr} = 0.00187 \quad \text{when } L_{pi} = 3150 \text{ mm.}$$

Here L_{pi} is the maximum possible dimension (test characteristic) of practical interest.

2. Derivation of a refined value C_{re} for the similar point of a beam (diameter 17 mm) with known precision, on the basis of the condition that the target function $\delta_{exp} - \delta_{cal} = 0$, by varying C_{pr} when $K = 1$. We find that $C_{re} = 0.602436$.

In Fig. 1a, we plot $\delta = f(L)$ according to the model in determining C and K by the first method. We find C_{re} with known precision at a single similar point 1 of the object of analysis and model, with scale factor $K = 1$ (Table 1).

The second method includes the following steps.

1. Preliminary determination of C_{pr} from the condition that $P(0, \dots, L_{pi}) = 0.9973$

$$C_{pr} = \ln 370.370/L_{pi} \quad \text{when } K = 1;$$

$$C_{pr} = 0.00187 \quad \text{when } L_{pi} = 3150 \text{ mm.}$$

2. Determination of K on the basis of the condition that the target function $\delta_{exp} - \delta_{cal} = 0$, for a single similar point. We find that $K = 10212.53$.

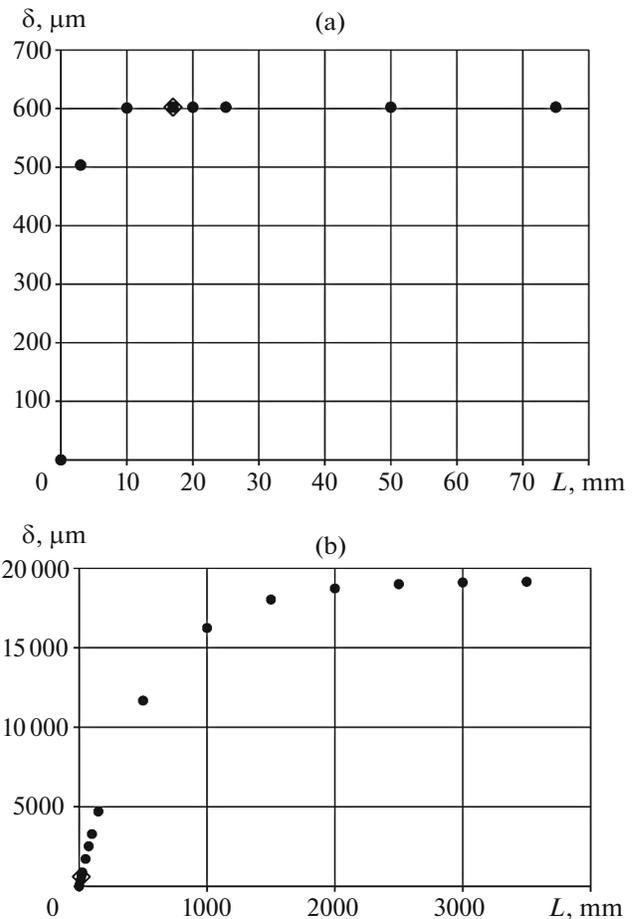


Fig. 1. Precision δ of dimension L determined by the first (a) and the second (b) methods: (a) $\delta_{cal1} = 602.4 \mu\text{m}$, $\delta_{exp1} = 602.4 \mu\text{m}$, $C = 0.6024$, $K = 1$; (b) $\delta_{cal1} = 602.4 \mu\text{m}$, $\delta_{exp1} = 602.4 \mu\text{m}$, $C = 0.001878$, $K = 10212.53$. (●) calculation; (◆) experiment.

In Fig. 1b, we plot $\delta = f(L)$ according to the model in determining C and K by the second method. We find $C_{re} = C_{pr} = 0.001878$ with $K = 10212.53$, in the case of known precision at similar point 1 of the object of analysis and the model (Table 1).

As an example of the practical use of the proposed probabilistic model, we consider the determination of C and K in certification of a 1A616 lathe with the goal of predicting the expected dimensional precision

Table 1. Precision values with a single similar point

Point	Machine tool	$\Delta_{exp}, \mu\text{m}$	L, mm	σ, mm	$P(L)$
1	1A616	602.40	17	0.1004	0.9973
2	1A616	748.10	20	0.1247	0.9973
3	1K62	602.40	17	0.1004	0.9973
4	1K62	882.36	25	0.1471	0.9973

Table 2. Confidence interval Δ and probability P with one-sided scattering field (3σ rule)

Range of σ	Δ , mm	$P(L)$	Dimension L , mm
σ	0.1004	0.021	17.1004
$\pm 2\sigma$	0.2008	0.157	17.2008
$\pm 3\sigma$	0.3012	0.498	17.3012
$\pm 4\sigma$	0.4016	0.839	17.4016
$\pm 5\sigma$	0.5020	0.960	17.5020
$\pm 6\sigma$	0.6024	0.997	17.6024

when the precision of the object of analysis is known at two similar points: $n = 2$.

Certification of equipment entails assessment of its distinctiveness and the individuality of the shaping conditions, taking account of the technological and operational characteristics, wear, and other details at the instant of machining. The distinctiveness of the shaping conditions is based on the degree of similarity C of the technological action in the model

$$\delta_{L_i} = \left[1 - e^{-(CL)}\right]CK.$$

First, we analyze the concept of the precision δ_{exp} at similar points of the object of analysis and the model. The precision δ_{exp} of dimension L is the width of the confidence interval $\Delta = L_{\text{max}} - L_{\text{min}}$

$$\Delta = \delta_{\text{exp}} = \pm 3\sigma = 6\sigma$$

with the corresponding confidence probability $P(L_i)$. For example, $P(L) = 0.9973$ according to the 3σ rule.

To determine the accuracy δ_{exp} of dimension L , we need to find its standard deviation σ . To that end, in certification, two batches of rollers 50 pcs. in each (diameter 17 and 20 mm) are preliminarily machined. For each batch, the standard deviation σ of dimension L is determined. Then, with the chosen confidence level, the width Δ of the confidence interval is determined from the 3σ rule (Tables 2 and 3).

If $P(L_i) = 0.9973$, we determine the width of the confidence interval $\Delta = \delta_{\text{exp}} = \pm 3\sigma$ or $\Delta = \delta_{\text{exp}} = 6\sigma$, respectively, for a symmetric or one-sided position of the scattering field relative to the rated value of L . In the present case, we find that $\sigma_1 = 0.1004 \mu\text{m}$ and $\sigma_2 = 0.12468 \mu\text{m}$ for similar points 1 and 2 (Table 1), respectively, when $L = 17$ and 20 mm for the rollers.

For the similar points, $\Delta_1 = \delta_{\text{exp}1} = 6\sigma = 602.4 \mu\text{m}$ and $\Delta_2 = \delta_{\text{exp}2} = 6\sigma = 748.1 \mu\text{m}$. With a confidence level $P(L) = 0.9973$, we conclude that 99.73% of the rollers of diameter 17 and 20 mm machined on the 1A616 lathe fall within these intervals. The possible dimensional precision, the position of the scattering field relative to the center of the group, and the width of the intervals for rollers of diameter 17 mm are shown in Fig. 2 (see also Tables 1–3).

Table 3. Confidence interval Δ and probability P with symmetric scattering field (3σ rule)

Δ , mm	L , mm	$P(L)$
$\pm\sigma$	17 ± 0.1004	0.6826
$\pm 2\sigma$	17 ± 0.2008	0.9544
$\pm 3\sigma$	17 ± 0.3012	0.9972

After preliminary determination of σ and Δ_{exp} in certification of the 1A616 lathe, as well as $P(L) = 0.9973$, we may calculate C and K .

Mathematically, if we know the dimensional precision δ_{exp} of the object of analysis and the model for at least two similar points ($n \geq 2$), we may determine C and K by solving the equations

$$\delta_{\text{exp}1} = [1 - \exp(-CL_1)]CK;$$

$$\delta_{\text{exp}2} = [1 - \exp(-CL_2)]CK;$$

...

$$\delta_{\text{exp}i} = [1 - \exp(-CL_i)]CK.$$

Note that, as a result of the errors in determining the set of $\delta_{\text{exp}i}$ values, the final C and K values are determined by the least squares method and correlation analysis in practice, as a rule. In other words, the sum of squares of the difference between $\delta_{\text{exp}i}$ and $\delta_{\text{cal}i}$ is minimized

$$\sum_{i=1}^n (\delta_{\text{exp}i} - \delta_{\text{cal}i})^2 \rightarrow \min.$$

Here the correlation coefficient

$$CC = f(\delta_{\text{exp}i}, \delta_{\text{cal}i})$$

has a maximum value: for example, $CC = 0.97$.

Note that correlation analysis permits refinement of the form of the relationship $\delta_{\text{cal}} = f(C, K, L)$ by determining C , while the least squares method permits minimization of the difference of $\delta_{\text{exp}i}$ and $\delta_{\text{cal}i}$ by determining K . Using the results of correlation analysis to determine C and K is only recommended when $n > 2$. With $n = 1$ and $n = 2$, refined values of C and K are only used on minimization of the target function

$$f\left(\sum_{i=1}^n (\delta_{\text{exp}i} - \delta_{\text{cal}i})^2\right) \rightarrow \min.$$

In the light of the foregoing, we may determine C and K as follows.

1. Preliminary determination of $C = C_{\text{pr}}$ from the condition that $P(0, \dots, L_{\text{pi}}) = 0.9973$ when $K = 1$.

2. Preliminary determination of the correlation coefficient CC_{pr} between the experimental δ_{exp} and calculated δ_{cal} values of the dimensional precision.

3. Refinement of C_{pr} by optimizing the target function $CC_{\text{pr}} = f(C, K, L)$ until $CC_{\text{pr}} = CC_{\text{max}} = 0.97$ or the

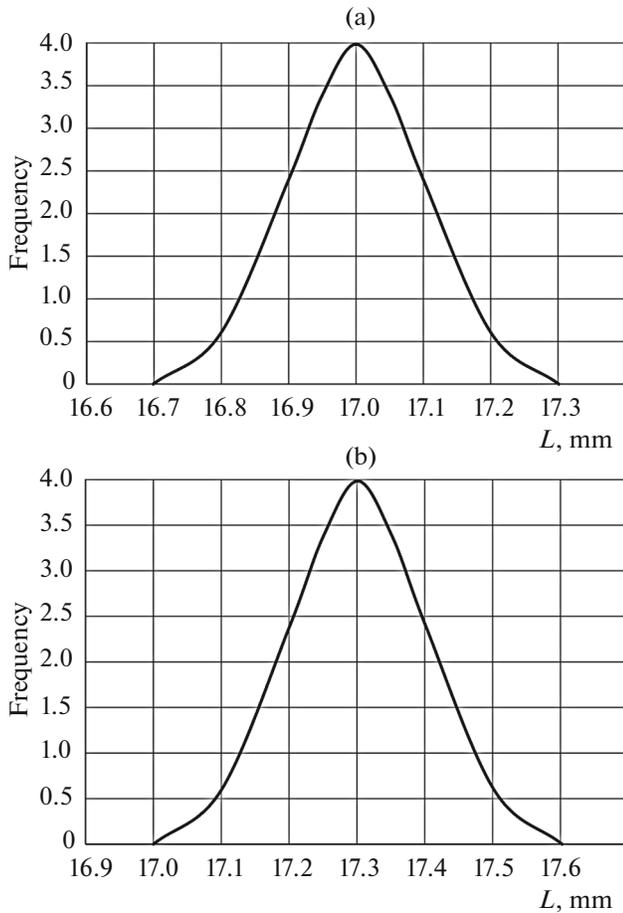


Fig. 2. Dimension L of a batch of rollers (diameter 17 mm) with a symmetric (a) and one-sided (b) scattering field.

target function $f\left(\sum_{i=1}^n (\delta_{\text{exp}i} - \delta_{\text{cal}i})^2\right) \rightarrow \text{min}$ with variation in C_{pr} .

4. Refinement of K by minimizing the target function $f\left(\sum_{i=1}^n (\delta_{\text{exp}i} - \delta_{\text{cal}i})^2\right) \rightarrow \text{min}$ on the basis of the least squares method.

Note also that, to ensure similarity of the model and the object of analysis—in other words, to ensure that $\delta_{\text{exp}i} = \delta_{\text{cal}i}$ when $n = 1$ and $n = 2$ —we consider two

Table 4. Calculation results

Approximation by means of	C	K	Target function $\Sigma(\delta_{\text{exp}} - \delta_{\text{cal}})^2$
K	0.014786	191.4750722	16823.48594
K	0.014786	191.4750722	16823.48594
C	0.675271	1	10614.88225

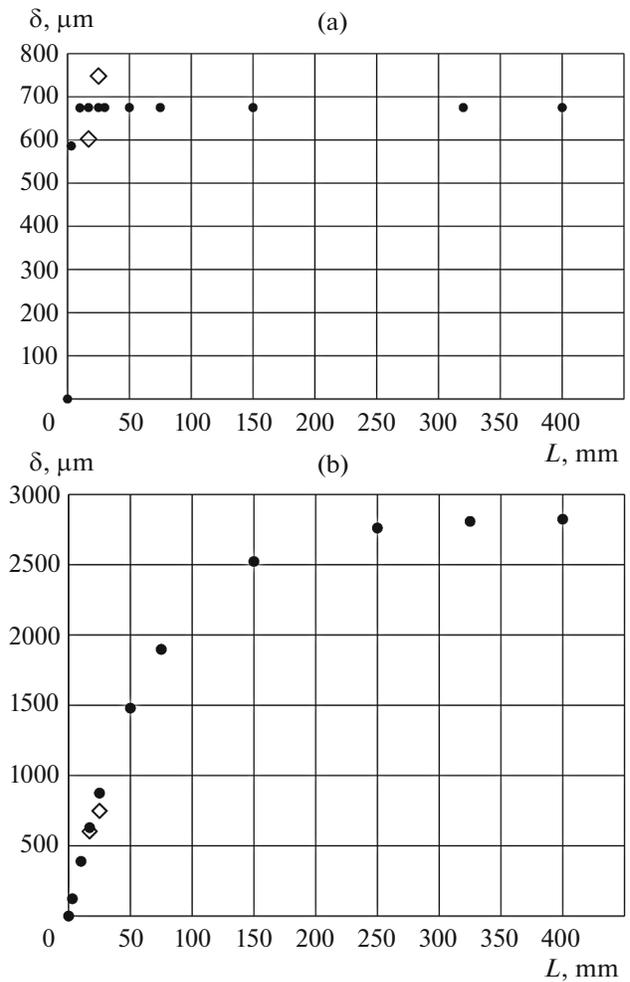


Fig. 3. Dimensional precision $\delta_{\text{cal}} = f(C, K, L)$ when $K = 1$ (a) and 191.5 (b): (a) $\delta_{\text{cal}1} = 675.26 \mu\text{m}$, $\delta_{\text{cal}2} = 675.27 \mu\text{m}$, $\delta_{\text{exp}1} = 602.41 \mu\text{m}$, $\delta_{\text{exp}2} = 748.12 \mu\text{m}$; (b) $\delta_{\text{cal}1} = 629.33 \mu\text{m}$, $\delta_{\text{cal}2} = 875.00 \mu\text{m}$, $\delta_{\text{exp}1} = 602.41 \mu\text{m}$, $\delta_{\text{exp}2} = 748.12 \mu\text{m}$.

approaches to determining C and K : (1) determination of K and subsequent correction of C ; (2) determination of C and subsequent correction of K .

We now determine C and K with two similar points 1 and 2, whose precision is obtained on a 1A616 machine tool for the maximum possible dimension $L_{\text{pi}} = 400 \text{ mm}$ in the conditions of practical interest, in machining rollers of diameter 17 and 20 mm when $\sigma_1 = 0.100402$, $\sigma_2 = 0.124688$, $\Delta_1 = \delta_{\text{exp}1} = \pm 3\sigma = 602.4147 \mu\text{m}$, $\Delta_2 = \delta_{\text{exp}2} = \pm 3\sigma = 748.1252 \mu\text{m}$, and $C_{\text{pr}} = \ln 370.370/L_{\text{pi}} = 0.014786$ with $K = 1$.

The results for C and K are shown in Table 4, while δ_{cal} is plotted in Fig. 3. Analysis shows that, on approximation of the similar points with variation in C , the calculated precision is less than with variation in K . However, the plot of $\delta_{\text{cal}} = f(C, K, L)$ when $K = 1$ (Fig. 3a) is much less clear than the plot of $\delta_{\text{cal}} =$

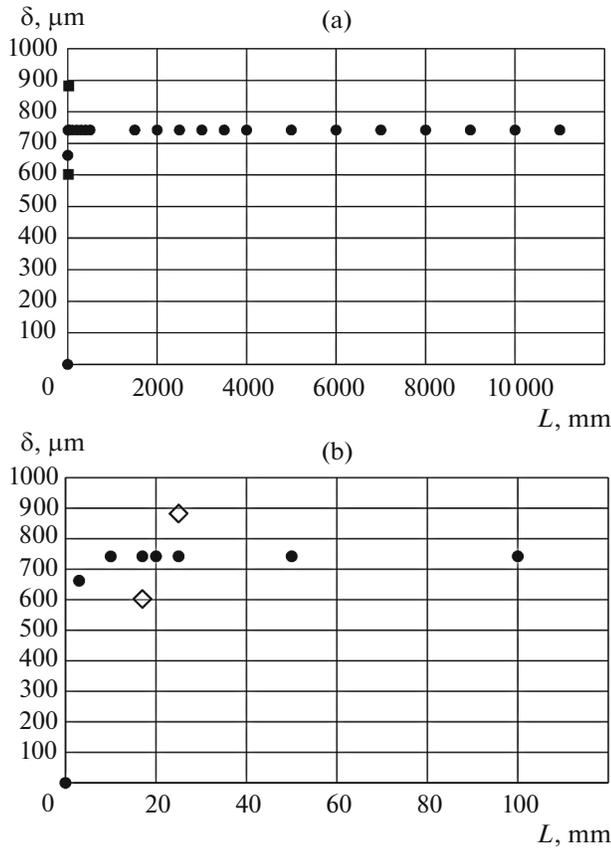


Fig. 4. Relative position of experimental and model points (a) and a fragment of the graph on an expanded scale (b).

$f(C, K, L)$ when $K = 191.5$ (Fig. 3b). The selection of the method of determining C and K depends on the specifics of the research; the researcher makes the final decision.

Theoretically, in certification of equipment, determination of C when $K = 1$ is possible if the precision is known for a single similar point. However, to improve

the accuracy of certification and to plot a satisfactory graph of $\delta = f(L)$, the use of values for several similar points is recommended.

We now consider the determination of C and K for two similar points 3 and 4 when $L_{pi} = 10000 \text{ mm}$ for rollers of diameter 17 and 25 mm when $\sigma_3 = 0.1004 \mu\text{m}$, $\sigma_4 = 0.1470596 \mu\text{m}$, $\Delta_{exp3} = \delta_{exp3} = \pm 3\sigma = 602.4 \mu\text{m}$, and $\Delta_{exp4} = \delta_{exp4} = \pm 3\sigma = 882.358 \mu\text{m}$. The model parameters are shown in Table 5.

In Fig. 4, we present the relative position of the similar points and the points in the model calculation with $C = 0.742378$ and $K = 1$. We determine C and K and also the value of the target function (39187.55603) with approximation of the similar points by varying C in the model equation.

In Fig. 5, we show the dimensional precision δ_{cal} in the range $0 < L_{pi} < 10000 \text{ mm}$ according to the model calculation. We determine $C = 0.00059145$ and $K = 101694.921$ and also the value of the target function (0.6535) with approximation of the similar points by varying K in the model equation.

We now consider the determination of C and K for the proposed model when $n \geq 2$. For the experimental precision of the object of analysis at the similar points, we employ empirical dimensional precision values for quality class 7 with the existing tolerances.

With the set of dimensional precision values at the similar points of the model and the object of analysis, we determine C and K from the dimensional precision in quality class 7 over the range 0–3150 mm, as follows.

1. Determination of the preliminary value C_{pr} from the condition

$$P(L_{pi}) = 1 - e^{-C_{pr}L_{pi}} = 0.9973,$$

where $L_{pi} = 3150 \text{ mm}$ is the maximum possible value of the dimension under consideration; and $C_{pr} = 0.0018776$.

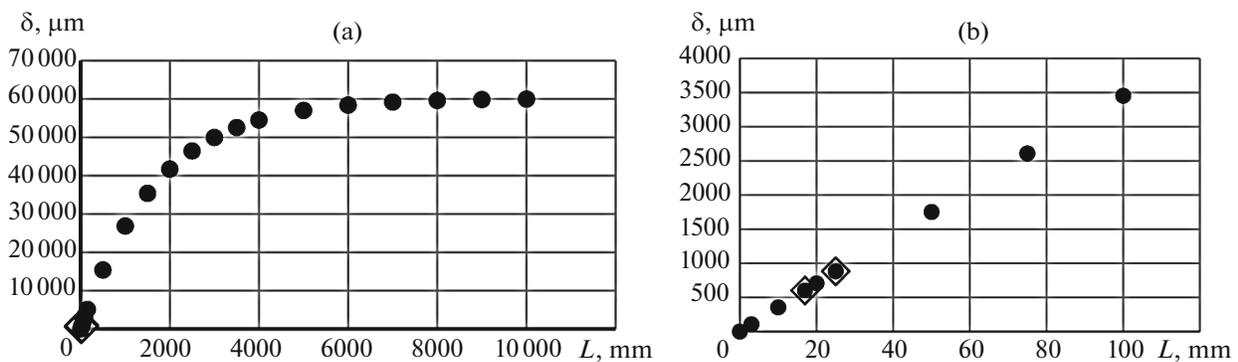


Fig. 5. Dependence of the precision δ_{cal} on L in the range $0 < L_{pi} < 10000 \text{ mm}$ (a) and a fragment of the graph on an expanded scale (b).

Table 5. Calculation results

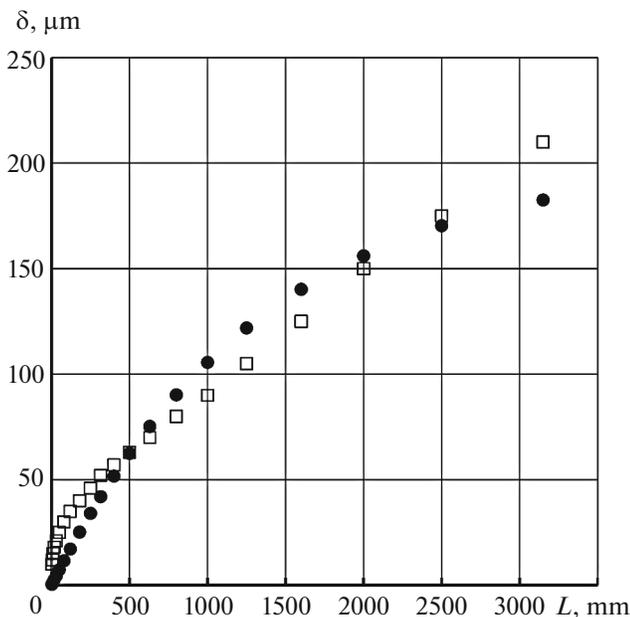
Approximation by means of	C	K	Target function $\Sigma(\delta_{\text{exp}} - \delta_{\text{cal}})^2$
K	0.000591	101694.9206	0.653522634
K	0.000591	101694.9206	0.653522634
C	0.742378	1	39187.55603
C	0.742378	1	39187.55603

2. Determination of the preliminary correlation coefficient CC_{pr} between the experimental and calculated values of the dimensional precision according to the model when $C = C_{\text{pr}}$ and $K = 1$ (Fig. 6). We find that $C_{\text{pr}} = 0.0018776$, $CC_{\text{pr}} = 0.9081564$ and $K = 1$.

3. Determination of the refined value C_{re} by optimization of the target function CC_{pr} with variation in C_{pr} until the maximum possible CC_{pr} value is obtained: for example, $CC_{\text{pr}} \rightarrow CC_{\text{max}} = 0.8-0.99$ when $K = 1$. We find that $C_{\text{re}} = 0.00073753$, $CC_{\text{max}} = CC_{\text{pr}} = 0.98$, and $K = 1$.

4. Refinement of K so as to minimize $\Sigma(\delta_{\text{exp}} - \delta_{\text{cal}})^2$ by the least squares method. We find that $K = 274.4282913$.

In Fig. 6, we show the final empirical results for the dimensional precision in quality class 7 with the existing tolerances and the precision calculated from the model with the given C and K values.

**Fig. 6.** Dependence of δ on L according to the model.

CONCLUSIONS

1. As a practical example, the coefficients C and K have been determined on the basis of the proposed model in certification of a 1A616 lathe with the goal of predicting the expected dimensional precision when the precision of the object of analysis is known at two similar points: $n = 2$.

2. On the basis of the C and K values obtained, we find that the accuracy given by the model in approximating the similar points is less with variation in C than with variation in K .

3. Theoretically, in certification of equipment, determination of C when $K = 1$ is possible if the precision is known for a single similar point. However, to improve the accuracy of certification and to plot a satisfactory graph of $\delta = f(L)$, the use of values for several similar points is recommended.

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CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

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