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Determining the Accuracy of a Product's Final Dimension by Simulation

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Abstract—A probabilistic approach to determining the final dimensional accuracy of a manufacturing product is considered. In this approach, the random action of many factors on the accuracy is taken comprehensively into account, regardless of the physicomechanical properties of the materials, the shaping method, and the characteristics of the metal-cutting tool. The method is based on simulation.

Keywords: manufacturing accuracy, scale factor, probabilistic model, simulation, functionally significant perturbations

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The structure of the model in simulation and the procedure for determining the input parameters were considered in [1–4]. On the basis of the test results, the statistical output characteristics of the model were calculated in accordance with probability theory and statistics [5, 6].

The output characteristics are determined as follows.

1. The test index (TI). In simulation, each test result is determined in accordance with the expression SC < CPP < L + SC. Here SC is the starting coordinate in simulation; and CPP is the coordinate of the physically significant technological perturbation in forming the final dimension L_i during the machining process. The test index is assigned the value 1 (or True) if there is a physically significant technological perturbation and the value 0 (or False) otherwise. A value of 1 corresponds to a single perturbation of the dimension L_i; and a value of 0 to no perturbation.

2. The probability of perturbation. The probability of perturbation of the dimension L_i in a series of tests (n_{tot} = 300) is determined from the formulas

$$P(L_i) = \sum \text{NP}_{\text{L}_\text{i}} / \sum \text{NT}_{L_i};$$

$$P(L_i) = \frac{\sum \text{NP}_{L_i}}{\sum \text{NT}_{L_i}} = \frac{\sum_{i=1}^{n_{\text{tot}}} \text{NT}_{L_i}}{n_{\text{tot}}},$$

where $\sum \text{NP}_{L_i}$ is the total number of perturbations of L_i in the series of tests; and $\sum \text{NT}_{L_i}$ is the total number of tests in the series corresponding to stabilization of the relative frequency of perturbations of L_i. Note that $0 \le \sum \text{NP}_{L_i} \le n_{\text{tot}}$ and $0 < L < 3500$.

The number of perturbations NP_{L_i} of the final dimension L_i corresponding to physically significant technological perturbations in simulation is determined from the condition SC < CPP < L + SC. If this condition is true, there is a perturbation of dimension L_i; otherwise, there is no perturbation.

We determine $\sum \text{NP}_{L_i}$ and $\sum \text{NT}_{L_i}$ from the formulas

$$\sum \text{NP}_{L_i} = \sum_{i=1}^{n_{\text{tot}}} \text{TI}_{L_i};$$

$$\sum \text{NT}_{L_i} = n_{\text{tot}}.$$

For all the L values considered in each series of tests, the total number is assumed to be n_{tot} = 300. That guarantees stabilization of the current value $P(L_i)_{\text{cu}}$ and $(\sum \text{PP}_i)_{\text{cu}}$, where PP denotes a physically significant technological perturbation. In this case, in other words, the calculated values are as close as possible to their mathematical expectation, and the mean square deviation is a minimum, according to [5–8].

3. The relative perturbation coefficient C_{L_i}. In simulating the formulation of the accuracy of L_i, the relative technological perturbation coefficient for a single test is determined from the formula

$$C_{L_i} = \frac{\sum \text{NP}_{L_i}}{n_{\text{tot}} L_i},$$

where i is the index of the specific final dimension.

In physical terms, C_{L_i} is the mean number of perturbations per unit length of L_i in a single test.

4. The mean relative perturbation coefficient $C_{\text{me.s}}$. The mean relative technological perturbation coefficient for a group of m final dimensions from the set L_i in a single test is determined from the formula

$$C_{\text{me.s}} = \left(\sum_{i=1}^{m} C_{L_i}\right)/m = \sum_{i=1}^{m}\left(\frac{\sum \text{NP}_{L_i}}{n_{\text{tot}} L_i}\right)/m,$$

where L_i is any positive integer.

Each dimension L_i is investigated in constant simulation conditions. In physical terms, $C_{\text{me.s}}$ is the mean number of perturbations per unit length of L_i in a single test, with constant simulation conditions, for a set of m different L_i values.

To define the accuracy of the final dimensions on the basis of probabilistic simulation, we consider the formation of the dimensional error [9–14].

1. The error is not formed continuously, but only at the instant of the physically significant technological perturbation. At that moment, the perturbation distorts the final dimension by the unit $\Delta\delta_i$ of accuracy.

2. The error is not formed at once, but as a sum of suberrors corresponding to each perturbation of the final dimension, with specified conditions in the series of tests.

3. We assume that the coefficient $C_{\text{me.s}}$—the mean relative technological perturbation coefficient for a group of m final dimensions from the set L_i in a single test (with fixed machining conditions)—is constant and is equal to the accuracy unit $\Delta\delta_i$ of the machining operation: $\Delta\delta_i = C_{\text{me.s}}$.

Under these assumption, for each of the m elements in the group of L_i values ($1 \le i \le m$), the error $\delta_s(L_i)$ is determined as the product of the accuracy unit $\Delta\delta_i$ (or the coefficient $C_{\text{me.s}}$) and the total number of perturbations $\sum \text{NP}_{L_i}$ for L_i in simulation

$$\delta_s(L_i) = \Delta\delta_i \sum \text{NP}_{L_i} = C_{\text{me.s}} \sum \text{NP}_{L_i}.$$

In determining $\sum \text{NP}_{L_i}$, we take account of the probability of perturbation in the final dimension

$$P_{L_i} = \sum \text{NP}_{L_i}/n_{\text{tot}},$$

where $\sum \text{NP}_{L_i} = P_{L_i} n_{\text{tot}}$. Then

$$\delta_s(L_i) = C_{\text{me.s}} P_{L_i} n_{\text{tot}}.$$

Let

$$C_{\text{me.s}} n_{\text{tot}} = C_s.$$

Then

$$\delta_s(L_i) = C_s P_{L_i}. \quad (1)$$

This is a probabilistic model for determining the accuracy of the dimension L_i in simulation. To adapt the model to a specific practical case, we determine the similarity coefficient K, on the basis of the following principles.

1. The dimensions of K are chosen so as to ensure that the dimensions of the comparable quantities of the object of analysis and the model are the same. In the present case, this means that the left and right sides of Eq. (1) will be equal.

2. The numerical value of K is such that the value of the experimental parameter and the value calculated from Eq. (1) are the same at the corresponding point or points—that is, points for which there are dimensional relations of the process parameters with a numerically identical value.

In the light of the foregoing, we specify the dimensional relation of the parameters for solution of our problem. To that end, we introduce the scale parameters S_s and S_{ob} for the model and the actual object, respectively, taking into account that $P(L) = 1 - e^{-CL}$. We find that

$$\delta_{\text{ob}} S_{\text{ob}} = \left(1 - e^{-CL}\right) C S_s;$$

$$\delta_{\text{ob}}[L] S_{\text{ob}}[1/L] = \left(1 - e^{-CL}\right) C[1/L] S_s[L];$$

$$S_s/S_{\text{ob}} = K[L^2];$$

$$\delta_{\text{ob}}[L] = \left(1 - e^{-CL}\right) C[1/L] K[L^2],$$

where S_s and S_{ob} are dimensional parameters for the model and the object; δ_{ob} and δ_s are the accuracy of the object and the simulation based on the model; K is the dimensional similarity coefficient of the model, mm^2; and the dimensions of the parameters in terms of the length L (mm) are given in the square brackets.

In Fig. 1, we show the dependence of the accuracy $\delta_s(L_i)$ and the probability $P_s(L_i)$ of perturbation of the final dimension in simulation on the dimension L_i.

In Eq. (1), we analyze the mean relative technological perturbation coefficient C_s. In physical terms, C_s is the mean number of perturbations per unit length of L_i over a series of tests n_{tot}—that is, over the whole cycle in which each dimension L_i is formed, in constant machining conditions. Thus, C_s corresponds to identical machining conditions or identical manufacturing operations at a specific workstation.

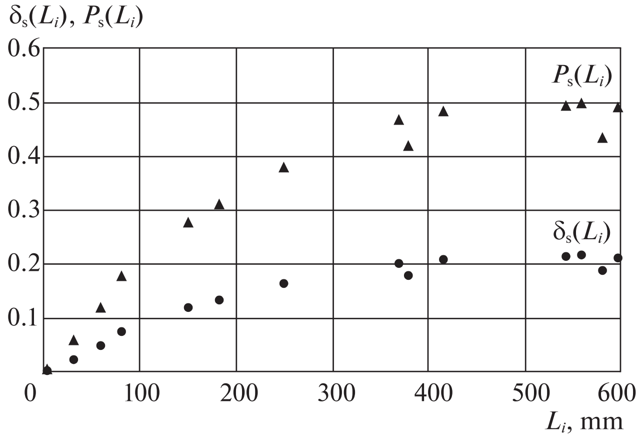

Fig. 1. Dependence of the accuracy $\delta_s(L_i)$ and probability $P_s(L_i)$ of perturbation on the final dimension L_i in simulation.

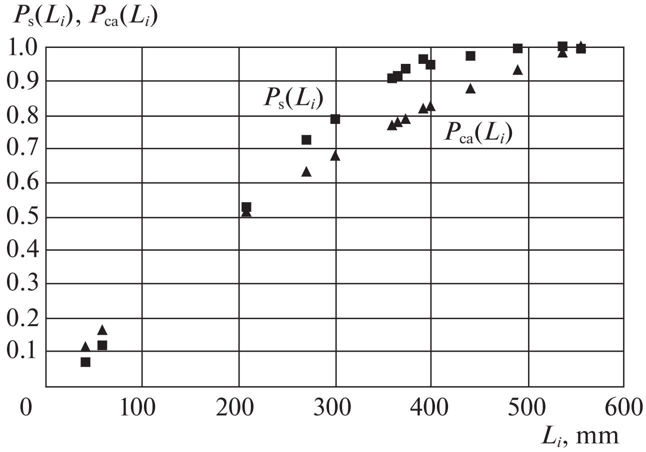

Fig. 2. Dependence of the normalized probabilities $P_s(L_i)$ and $P_{ca}(L_i)$ of perturbation on L_i.

Taking into account that $\delta_s(L_i) = C_s P_{L_i}$, while $C_s = C_{me.s} n_{tot}$, we may write

$$C_{me.s} = \sum_{i=1}^{m} \left(\frac{NP_{L_i}}{n_{tot} L_i} \right) / m;$$

$$\delta_s(L_i) = P_{L_i} n_{tot} \sum_{i=1}^{m} \left(\frac{NP_{\Sigma L_i}}{n_{tot} L_i} \right) / m$$

or

$$\delta_s(L_i) = P_{L_i} \sum_{i=1}^{m} \left(\frac{NP_{\Sigma L_i}}{L_i} \right) / m.$$

The actual final dimension is

$$L_{aci} = L_i \pm (L_i)/2$$

or

$$L_{aci} = L_i \pm \delta_s(L_i)/2 = L_i \pm P_{L_i} n_{tot} C_{me.s}/2.$$

Hence

$$L_{ac} = L_i P_{L_i} C_s/2.$$

In scientific research, it is often necessary to determine the relation between outcomes and variable factors. Correlation analysis has been developed for that purpose.

A correlation is useful when a quantity corresponds to several other quantities. The correlation may be linear or nonlinear. In a linear correlation, equal changes in one quantity correspond to equal changes in the mean values of the other quantities. In a nonlinear correlation, equal changes in one quantity correspond to unequal changes in the mean values of the other quantities.

A correlation may be direct or inverse. In a direct correlation, increase in one quantity is associated with increase in the mean value of another quantity. The corresponding correlation coefficient is positive. In an inverse correlation, increase in one quantity is associated with decrease in the mean value of another quantity. The corresponding correlation coefficient is negative.

A correlation may be represented by tables, a graph, or a coefficient (or ratio). In a correlation graph, the coordinates of the points are quantities corresponding to a single object. Tables and graphs provide only a general idea of the presence and direction of the relationships. Only the correlation coefficient permits measurement and assessment of their statistical reliability.

The correlation coefficient is a number corresponding to the strength and direction of the relation between the given random quantities. In speaking of the correlation coefficient, we almost always mean the Pearson correlation coefficient, which may only be calculated if the relation between the variables is linear and the variables are normally distributed.

In the present work, by investigating the formation of the final dimension on the basis of simulation and analysis, we determine the theoretical probability from the expression $P(L) = 1 - e^{-CL}$ and also the probability $P_s(L_i)$ derived by simulation (Fig. 2). We find that the final dimension $R = 0.98$ (Fig. 3).

We use the traditional method of correlation analysis [2]. On the basis of the correlation coefficient, we judge the correctness or error of our hypothesis: that the dependence $P_s(L_i)$ is exponential in simulation. In the analytical calculations, we assume that the coefficient is equal to the mean relative perturbation coefficient C_s over a cycle or a series of tests n_{tot}—that is, the coefficient of identical technological perturbations (CIP) determined in simulation [4].

Excel software is used for correlation analysis. After preliminary determination of $P_s(L_i)$ and $P_{ca}(L_i)$, the

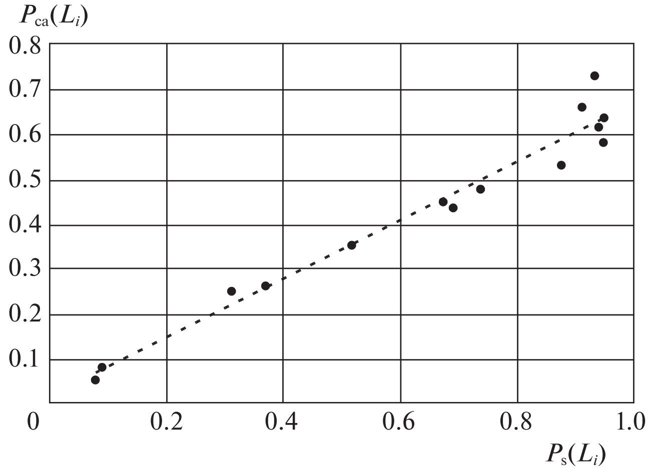

Fig. 3. Correlation of the probabilities of perturbation of the final dimension L_i obtained by simulation and analysis.

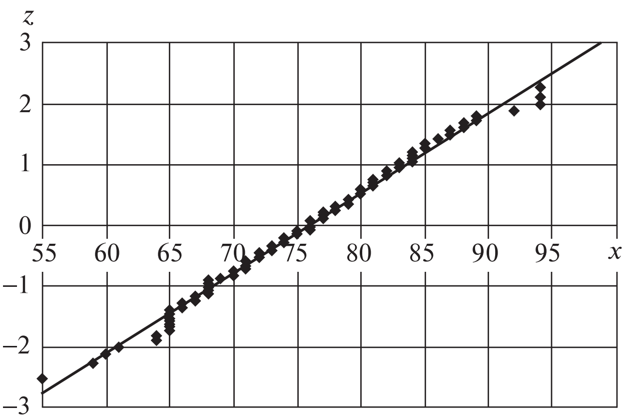

Fig. 4. Verification of a normal distribution.

corresponding data sets are analyzed so as to determine the correlation equation and correlation coefficient (Fig. 3).

In machining a batch of workpieces on a machine tool, the true dimension of each workpiece may be any random quantity within a certain interval, as a result of random factors.

Research by Yakhin, Zykov, and others shows that the distribution of the actual workpiece dimensions in a fixed machine-tool setup is often normal (Gauss's law). This may be explained by a familiar principle of probability theory: the distribution of the sum of a large number of mutually independent random terms (if each one has negligible and approximately equal influence on the sum and no dominant factors are present) is normal (Gaussian).

For the sake of expediency, monitoring data for the machining process should be expressed as a theoretical distribution as similar as possible to the experimental distribution. The hypothesis regarding the form of the distribution is verified by the Pearson, Kolmogorov, and other tests. Most often, the Pearson statistic χ^2 is used. However, large quantities of data are required for that purpose. For example, for the Pearson test, data sets no smaller than 50–100 are recommended. Therefore, with small samples, the hypotheses regarding the form of the distribution are tested by approximate methods—graphically or on the basis of the skew and kurtosis.

To verify the hypothesis of a normal distribution, we will use the experimental results from three repeated series (each with 56 tests), for a test dimension $L = 100$, using MS Excel functions.

Graphical analysis is the simplest method of assessing compliance with a specific distribution, but it is very approximate. The results are arranged in a variational series. Then for each result x_i, the accumulated frequency is calculated from the formula

$$W(i) = i/(n+1),$$

where i is the number of the result in the variational series; and n is the sample size.

If we use the accumulated frequencies as the distribution functions, the corresponding quantile of the presumed distribution may be found for each $W(i)$. In particular, for a normal distribution, we find the quantile of the standard normal distribution z_i, which is calculated from the MS Excel NORMSINV statistical function. The results are shown in Fig. 4 in the coordinates x and z, on the assumption that the quantities x_i are quantiles of the same distribution as z_i. In that case, there should be a linear relation between x and z. If the points deviate only slightly from a straight line, we assume that the results are satisfactorily described by the chosen theoretical distribution. With large deviations from linearity, the experimental distribution does not correspond to the chosen theoretical curve. In Fig. 4, the points lie close to a straight line, and so we accept the hypothesis of a normal distribution.

We may also use the skew and curtosis to approximately verify the hypothesis of a normal distribution.

The skew reflects the degree of asymmetry of the differential curve of the experimental distribution with respect to the differential curve for a normal distribution. It is calculated from the formula

$$A \approx \frac{1}{ns^3}\sum_{i=1}^{n}(x_i - \bar{x})^3.$$

The kurtosis reflects the elevation of the differential curve of the experimental distribution with respect to the differential curve for a normal distribution. It is calculated from the formula

$$E \approx \frac{1}{ns^4}\sum_{i=1}^{n}(x_i - \bar{x})^4 - 3.$$

Statistical functions are built into Excel software for the calculation of A (the SKEW function) and E (the KURT function).

For random values of A and E, their dispersions may be calculated from the formulas

$$D(A) = \frac{6(n-1)}{(n+1)(n+3)};$$

$$D(E) = \frac{24(n-2)(n-3)n}{(n-1)^2(n+3)(n+5)}.$$

The results obtained are as follows

$$A = 0.176573; \quad E = 0.293476; \quad D(A) = 0.034672;$$

$$\text{and} \quad D(E) = 0.133856.$$

If $|E| \le 5\sqrt{D(E)}$ and $|A| \le 3\sqrt{D(A)}$, the distribution is assumed normal. But the hypothesis is rejected if $|E| \gg \sqrt{D(E)}$ and $|A| \gg \sqrt{D(A)}$.

In the present case, we obtain the results

$$0.293476 \le 1.829316812 \quad \text{for } E;$$

$$0.176576 \le 0.558612567 \quad \text{for } A.$$

We see that the distribution may be assumed normal.

CONCLUSIONS

On the basis of simulation, we have considered the accuracy of the final dimension in machining. The outcome of the simulation—the accuracy of the final dimension—is found to have a normal distribution, while the graphical dependence $P(L_i) = f(L_i)$ increases nonlinearly, at a rate that gradually slows with increase in L_i. When using two methods—simulation and analysis—high correlation of the results is obtained. The correlation coefficient is $R = 0.98$ (Fig. 3). That indicates reliability of the results.

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REFERENCES

1. Barzov, A.A., Denchik, A.I., and Tkachuk, A.A., Simulation of the process of probabilistic formation of the executive size, *Nauka Tekh. Kazakhstana*, 2020, no. 1, pp. 39–47.
2. Tkachuk, A.A., Denchik, A.I., and Barzov, A.A., The probabilistic nature of the formation of the executive size during machining, *Materialy Mezhdunarodnoi nauchnoi konferentsii "XX Satpaevskie chteniya"* (Proc. Int. Sci. Conf. "XX Satpaev Readings"), 2020, vol. 17, pp. 377–384.
3. Barzov, A.A., Denchik, A.I., Musina, Zh.K., et al., Development of analytical model of probable formation of accuracy of executive size taking into account the influence of the scale factor, *Nauka Tekh. Kazakhstana*, 2021, no. 1, pp. 19–29.
4. Tkachuk, A.A., Denchik, A.I., and Barzov, A.A., Methodology for evaluating technological methods (technological processes) for guaranteed dimensional processing, *Materialy Mezhdunarodnoi nauchnoi konferentsii "XII Toraigyrovskie chteniya"* (Proc. Int. Sci. Conf. "XX Toraigyrov Readings"), 2020, vol. 6, pp. 103–109.
5. Popov, A.M. and Sotnikov, V.N., *Teoriya veroyatnostei i matematicheskaya statistika: Uchebnik i praktikum* (Probability Theory and Mathematical Statistics: Manual and Workshop), Moscow: Yurait, 2020, 2nd ed.
6. Sidnyaev, N.I., *Teoriya veroyatnostei i matematicheskaya statistika: Uchebnik* (Probability Theory and Mathematical Statistics: Manual), Moscow: Yurait, 2019.
7. Emel'yanov, S.G., Shvets, S.V., Remnev, A.I., et al., *Teoriya rezaniya: Matematicheskoe modelirovanie i sistemnyi analiz: Monografiya* (Cutting Theory: Mathematical Modeling and System Analysis: Monograph), Staryi Oskol: TNT, 2020.
8. Smirnov, V.A., *Matematicheskoe modelirovanie v mashinostroenii v primerakh i zadachakh: Uchebnoe posobie* (Mathematical Modeling in Mechanical Engineering in Examples and Tasks: Manual), Staryi Oskol: TNT, 2021.
9. Barzov, A.A., Denchik, A.I., Korneeva, V.M., et al., Probabilistic model of interaction of necessary and sufficient conditions for mass morbidity of the population, taking into account the scale-population factor materials, *Kachestvo Zhizn'*, 2020, no. 3 (27), pp. 19–26.
10. Grubyi, S.V., Simulation modeling of the cutting and tool wear processes, *Russ. Eng. Res.*, 2007, vol. 27, pp. 433–439. https://doi.org/10.3103/S1068798X07070064
11. Barzov, A.A., Galinovskii, A.L., Puzakov, V.S., et al., *Veroyatnostnoe modelirovanie v innovatsionnykh tekhnologiyakh* (Probabilistic Modeling in Innovative Technologies), Moscow: NT, 2006.
12. Epifanov, V.V., Relationship between the characteristics of machine parts and parameters of technological equipment, *Vestn. Mashinostr.*, 2021, no. 5, pp. 31–36. https://doi.org/10.36652/0042-4633-2021-5-31-36
13. Barzov, A.A., Denchik, A.I., Prokhorova, M.A., et al., *Masshtabnyi faktor* (*fenomenologiya i fiziko-tekhnicheskie polozheniya*) (Scale Factor (Phenomenology and Physico-Technical Provisions)), Moscow: Lomonosov Moscow State Univ., 2021.
14. Barzov, A.A., Belov, V.A., and Denchik, A.I., Information analysis of combined ultra-jet express diagnostics of materials and products of RST (rocket & space technology), *AIP Conf. Proc.*, 2019, vol. 2171, no. 1, p. 170014.
15. Dudak, N.S., Itybaeva, G.T., Taskarina, A.Zh., et al., A new pass-through lathe cutter, *Russ. Eng. Res.*, 2014, vol. 34, no. 11, pp. 705–707. https://doi.org/10.1007/s12541-017-0170-9
16. Mukanov, R.B., Bykov, P.O., Itybaeva, G.T., et al., Face turning of holes, *Russ. Eng. Res.*, 2019, vol. 39, no. 1, pp. 75–78. https://doi.org/10.3103/s1068798x19010064

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