Validating the First Law of Thermodynamics for Unsteady Flow in a Compression Wave Using Mathcad

DOI: 10.1615/HeatTransRes.2023051072

Vladimir Ryndin
Amangeldy Karmanov
Akmaral Kinzhibekova
Rizagul Dyussova
Gulnara Abdullina

ABSTRACT

Classical thermodynamics traditionally overlooks the role of quantities dependent on spatial coordinates and time, especially in the context of unsteady flows. This research introduces the first law of thermodynamics (FLT) tailored for non-stationary flow, distinguishing itself with the inclusion of terms bearing partial derivatives of pressure, p(x, t), concerning coordinates and time ($-\upsilon(\partial p/\partial x)dx$; $-\upsilon(\partial p/\partial t)dt$). By employing this novel approach, the derived equations are validated using a centred compression wave as a representative non-stationary flow case study. A methodology is also presented for experimentally quantifying hydrodynamic energy losses in the intake and exhaust systems of internal combustion engines. Central to the exploration is the calculation of pressure forces' work $-\upsilon(\partial p/\partial x)dx$ and $-\upsilon(\partial p/\partial t)dt$) in the FLT equation for non-stationary flows, particularly their applicability to a centred compression wave. Moreover, a distinct procedure for discerning friction work in non-stationary flow is delineated. The research methods encompass both analytical derivation and numerical simulations leveraging Mathcad software. The bespoke Mathcad program crafted for this study can graphically represent multiple flow parameters as functions of time, proving invaluable for Comprehending compression wave dynamics and evaluating friction work in diverse non-steady flows. Ultimately, the incorporation of energy equations tailored for non-stationary flows into classical thermodynamics paves the way for a more comprehensive understanding and application of thermodynamics to intricate flow scenarios

Keywords: hydrolosses; pressure recording; internal combustion engine; three-dimensional pressure; energy equation.

1. Introduction

Non-stationary (unsteady) flow occurs in the intake and exhaust systems of internal combustion engines and compressors. Calculating these flows is essential as the gas exchange processes influence cylinder filling and purging. Complex mathematical programs are employed for calculating the parameters of non-stationary flows. In well-known thermodynamics textbooks, the equation of the first law of thermodynamics (FLT) for non- stationary flow is not provided (Callen, 1985; Hołyst and Poniewierski, 2012; Kirillin et al., 1983; Lavenda, 2010; Morales-Rodriguez, 2016; Srivastava et al., 2019). In certain works, the energy equation for a non- stationary flow process is presented for an open system (control volume), where accounting for non-stationarity is achieved by considering various mass flows of incoming and outgoing substances (Borgnakke and Sannag, 2009; Krutov et al., 1991; Moran et al., 2014; Kondepudi and Prigogine, 2015). The energy equation for a moving element of non- stationary flow is not addressed here, either. On the other hand, I.I. Novikov (1984) examines the Euler equation, which contains partial derivatives of pressure with respect to coordinates and in the case of one-dimensional flow takes the form: $\rho\, dw/dt = -\,\partial p/\partial x$. This partial derivative is not computed further and is disregarded in the case of steady flow ($dp = (\partial p/\partial x)dx$). In the monograph by H.D. Baehr and K. Stephan (1973), an expression is provided for calculating the work of pressure forces through partial derivatives with respect to coordinates in steady flow (1):

$$dW^p = -\Delta V(\frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial y}dy + \frac{\partial p}{\partial z}dz) = -\Delta V dp. \tag{1}$$

It should be noted that the application of partial derivatives with respect to coordinates allowed Baehr to consider the specific work ($-\upsilon dp$) as the work of pressure forces associated with the displacement of an element of the medium as a whole. However, other authors interpret the meaning of this work differently, giving it various names ("available work," "technical work," "useful work", "actual work," "flow work"), which creates certain difficulties in the study of thermodynamics. The analysis of this issue is the focus of the work by V.V. Ryndin (1987; 1991). Thus, in classical thermodynamics, when formulating energy equations for flows, partial derivatives with respect to coordinates and time are not employed. This means that energy equations for non-stationary flows that arise in internal combustion engines and compressors are not presented. This is due to both the absence of such equations in thermodynamics textbooks and the complexity of computing integrals involving such derivatives.

2. Materials and Methods

In order to apply the laws of equilibrium thermodynamics to processes occurring within a moving medium, a small element of this medium ("macroparticle" or "liquid particle") is isolated in the form of a parallelepiped. The movement of this element is then considered under the influence of various forces. While the volume of the macroparticle may change during its motion, its mass remains constant. This signifies a closed equilibrium thermodynamic system, whose state is uniquely determined by the macroscopic parameters of pressure (p), velocity (υ), and temperature (T).

To evaluate integrals in these equations, both analytical derivation and numerical methods using the mathematical software Mathcad are employed. The analytical method involves deriving and proving expressions for partial derivatives and the corresponding works. However, it requires careful attention and substantial time investment, and it is not well-suited for visually representing the obtained functions as graphs. These difficulties can be avoided by performing calculations numerically using the Mathcad software. Utilizing Mathcad allows incorporating experimental pressure curves over time in the calculations, enabling immediate verification of the accuracy of derived dependencies for partial derivatives with respect to coordinates and time. It also facilitates the creation of graphs illustrating these dependencies, making the calculation method intuitive and practical.

The purpose of the article is approbation the energy equations (11) and (12) through their application in calculating the parameters of non-stationary flow, arising in a centred compression wave (pressure wave). Additionally, the article aims to develop a methodology for calculating friction work in non-stationary flow using the Mathcad software.

3. Results

3.1. Derivation of energy equations for non-stationary flow

The method of deriving the FLT equation presented below can be called "thermodynamic", since it does not include tensor calculus when calculating the work of viscous forces, as is customary in fluid and gas mechanics. In the general case, a liquid macroparticle is affected by spatial (mass, or volume) and surface forces: gravity, $\vec{F}_g$, pressure, $\vec{F}_p$, friction and forces acting from the side of the flow on the surface of a body immersed in it, or the reaction of the body to the flow (forces arising on the turbine blades and compressors are called technical, $\overrightarrow{F}_{tech}$ as they perform useful work in technical devices). If a force acts on the left side of the parallelepiped $p\delta y\delta z\vec{\imath}$(Figure 1) in the direction of the x-axis, then a force acts on the right side $-(p + \frac{\partial p}{\partial x}\delta x)\delta y\delta z\vec{\imath}$ in the opposite direction.

The component of the resulting of forces of pressure in the direction of the x-axis is determined as the sum of these forces (2):

$$\vec{F}_{p,x} = p\delta y\delta z\vec{\imath} - (p + \frac{\partial p}{\partial x}\delta x)\delta y\delta z\vec{\imath}. \qquad (2)$$

Taking into account the other two components $\delta\vec{F}_{\mathrm{p,\,y}}$ and $\delta\vec{F}_{\mathrm{p,\,z}}$ the resulting pressure force is determined as follows (3):

$$\delta\vec{F}_p = -\left(\frac{\partial p}{\partial x}\vec{\imath} + \frac{\partial p}{\partial y}\vec{\jmath} + \frac{\partial p}{\partial z}\vec{k}\right)\delta V = -\mathrm{grad}\, p\delta V, \qquad (3)$$

where $\vec{\imath}$, $\vec{\jmath}$, $\vec{k}$ – unit vectors of rectangular coordinate axes.

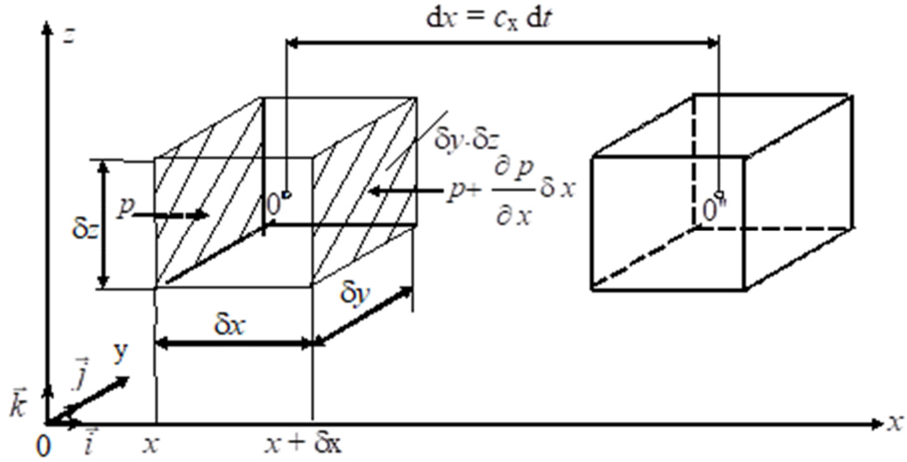


Figure 1. To the calculation of the work of pressure forces on the movement of an element of a moving medium

The work of the resulting of forces of pressure is defined as the scalar product of the vector of the resulting pressure forces applied at the centre of mass of the macroparticle, by the elementary displacement of the particle $\mathrm{d}\vec{r} = \mathrm{d}x\vec{\imath} + \mathrm{d}y\vec{\jmath} + \mathrm{d}z\vec{k}$ (4)

$$\delta^2 W_p = \delta\vec{F}_p \cdot d\vec{r} = -(\frac{\partial p}{\partial x}\mathrm{d}x + \frac{\partial p}{\partial y}\mathrm{d}y + \frac{\partial p}{\partial z}\mathrm{d}z)\delta V. \qquad (4)$$

If a particle moves in a uniform gravitational field $\delta\vec{F}_g = \vec{g}\delta m = -\vec{k}g\delta m$ (here it is assumed that the free-fall acceleration vector $\vec{g}$ is parallel to the z- axis and directed in the opposite direction), $\vec{g} = -\vec{k}g$, then the work of gravity will be equal to $\delta^2 W_g = \delta\vec{F}_g * d\vec{r} = -g\delta m dz$. Taking into account the remarks made, the law of change in the kinetic energy of a macroparticle can be written as follows (c is the velocity of a macroparticle) (5):

$$\delta m dc^2/2 = -(\frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial y}dy + \frac{\partial p}{\partial z}dz)\delta V - g\delta m dz - \delta^2 W_{\mathrm{fr}} - \delta^2 W_{\mathrm{tech}}, \qquad (5)$$

where the friction work $\delta^2 W_{\mathrm{fr}} > 0$, and the minus sign indicates that the friction forces are always directed opposite to the direction of movement; technical work $\delta^2 W_{\mathrm{tech}}$ in the turbine is considered positive, and in the compressor – negative).

Dividing all the terms of the last equation by the mass of the macroparticle, $\delta m = \rho\delta V = \delta V/\upsilon$, where υ is the specific volume, the balance of specific work (6):

$$d\mathrm{c}^2/2 = -\upsilon(\frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial y}dy + \frac{\partial p}{\partial z}dz) - gdz - \delta w_{\mathrm{fr}} - \delta w_{\mathrm{tech}}. \qquad (6)$$

Equation (6) expresses the law of change in the kinetic energy of the orderly motion of a fluid element of unit mass. Adding (6) with the equation of the first law of thermodynamics, $\mathrm{d}u = \delta q - p\mathrm{d}v$, which expresses the law of change in the potential and kinetic energies of the chaotic motion of microparticles of the medium relative to the centre of mass of the macroparticle, we obtain the equation of the first law of thermodynamics for non-stationary flow (7):

$$d\,u + dc^2/2 = \delta q - pdv - v(\frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial y}dy + \frac{\partial p}{\partial z}dz) - gdz - \delta w_{\mathrm{fr}} - \delta w_{\mathrm{tech}}. \quad (7)$$

Taking into account the obvious relations for pressure differentials (8) and enthalpy (9):

$$dp = dp(t, x, y, z) = \frac{\partial p}{\partial t}dt + \frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial y}dy + \frac{\partial p}{\partial z}dz, \quad (8)$$

$$dh \;=\; du + d(pv), \quad (9)$$

and also, the fact that, in the general case, heat δq is the sum of external heat δq^{e} and friction heat $\delta q_{\mathrm{fr}} = \delta w_{\mathrm{fr}}$, equation (9) can also be given the following form (10):

$$\delta q^{\mathrm{e}} = dh + \frac{dc^2}{2} + gdz - v\frac{\partial p}{\partial t}dt + \delta w_{\mathrm{tech}}. \quad (10)$$

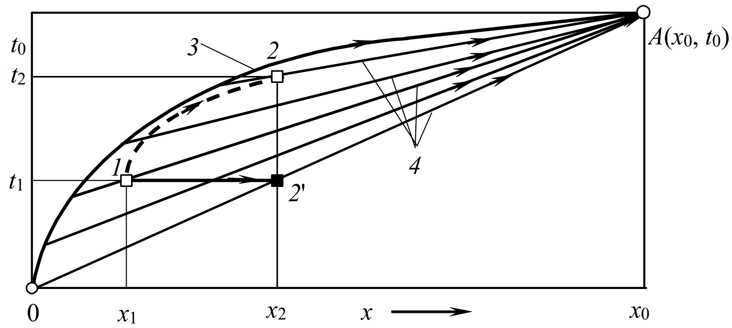
In the case of a stationary flow ($\mathrm{d}p/\mathrm{d}t = 0$ and $\mathrm{d}p = \mathrm{d}p(x, y, z) = \frac{dp}{dx}dx + \frac{dp}{dy}dy + \frac{dp}{dz}dz$), equations (6, 7, 10) acquire a well-known form. In the case of a non-stationary energy-isolated one-dimensional isentropic gas flow (we take $g\mathrm{d}z = 0$), the energy equations (6) and (10) in the integral form will take the following form (11, 12):

$$c_2^2/2 - c_1^2/2 = -\int_1^2 v\frac{\partial p}{\partial x}dx \equiv w_{\mathrm{conv}}, \quad (11)$$

$$\frac{a_1^2}{k-1} - \frac{a_2^2}{k-1} + c_1^2/2 - c_2^2/2 = -\int_1^2 v\frac{\partial p}{\partial t}dt \equiv w_{\mathrm{loc}}. \quad (12)$$

3.2. Centred compression wave

A centred compression wave can be obtained in the pipe in front of the piston when it moves according to a certain law, when all the characteristics (straight lines along which the gas parameters remain constant) intersect at one point $A(x_0, t_0)$ (Figure 2). The coordinate of point A can be determined by the formula $x_0 = a_{\mathrm{n}} t_0$, where a_{n} is the speed of sound in an undisturbed gas, when the flow velocity is zero with $c_{\mathrm{n}} = 0$. In the case of an accelerated movement of the piston, a shock wave is formed from the compression wave at point A. From the moment of formation of the shock wave, the equations describing the isentropic flow cease to be valid, since after the appearance of the shock wave, the entropy of each particle of the medium changes.



1-2 – macroparticle trajectory; *1-2'*– streamline;
3 – piston path; *4* – characteristics.

Figure 2. Centred compression wave

At any point of the *xt* plane, the direction of the characteristic of the first (second) family or C^+ (C^-) characteristics is determined by the equation $dx/dt = c \pm a$ (Kraiko, 2007). As applied to a compression wave, the main characteristic equation has the form (13) (Stanyukovich, 1971):

$$\frac{x-x_0}{t-t_0} = \mathrm{c} + a = a_\mathrm{n} + \frac{k+1}{2} * c. \tag{13}$$

Equation (13) is used to find the flow velocity *c* and the speed of sound *a*.

3.3. Calculation verification in the Mathcad package of FLT equations for unsteady flow

All calculations, including the calculation of integrals in equations (11) and (12), are performed using the Mathcad package (system). The use of programming in the Mathcad package is motivated by several reasons. Unlike other programming languages, the program's algorithm and the Mathcad program itself are written in the same symbols. This reduces both the program's writing volume and makes it visually clear and ready for direct editing. Literature on the usage of the Mathcad system is extensive (hundreds of publications). As an example, the works D.V. Kiryanov (2006), E.G. Makarov (2011), G.Ch. Shushkevich and S.V. Shushkevich (2010) can be mentioned. The use of other programming languages does not allow seeing calculation results at every moment during computations and constructing graphical dependencies of the obtained results.

Everything written below, including comments, can constitute the content of the calculation program. The system itself determines where the text is and where the mathematical expressions are (for clarity, in the textual part, symbols of quantities will be written in italics as commonly accepted, and in Mathcad formulas – in vertical font). The input of the assignment symbol with an equal sign ":=" is done by pressing the key with the colon symbol ":". Subscripts specifying quantities are placed after pressing the key with the dot.

Input of initial data: point coordinates: $\mathrm{x_1}$:=0.8 m; $\mathrm{x_2}$:=1 m; $\mathrm{x_0}$:=10 m; $\mathrm{a_n}$:=340 m/s – speed of sound in undisturbed gas; k:=1.4 – adiabatic index for air; R:=287 J/(kg·K) – specific gas constant of air; $\mathrm{p_n}$:=101325 Pa – pressure of undisturbed gas. Calculation of gas parameters in the undisturbed state according to the coordinates in Figure 2 yielded the following result:

- $\upsilon_\mathrm{n} := \frac{\mathrm{a_n^2}}{\mathrm{k \cdot p_n}} = 0.815$ m³/kg – specific volume of undisturbed gas;
- $\mathrm{T_n} := \frac{\mathrm{a_n^2}}{\mathrm{k \cdot R}} = 287.7$ K – temperature of the undisturbed gas;
- $\mathrm{t_0} := \frac{\mathrm{x_0}}{\mathrm{a_n}} = 0.029412$ s – time before the formation of a pressure jump;
- $\mathrm{t_1} := \frac{\mathrm{x_2}}{\mathrm{a_n}} = 0.002941$ s – time at point *1*, equal to time $t_{2'}$.

From equation (13), the flow velocity and the speed of sound in section *1* (the output for reference of the entered values: $\mathrm{x_1}$= 0.8 and $\mathrm{t_1}$= 0.002941 is carried out by pressing the key with the

"=" symbol) (14, 15):

$$c_1 := \frac{2}{k+1}\left(\frac{x_1 - x_0}{t_1 - t_0} - a_n\right) = 6.296\ \mathrm{m/s}, \tag{14}$$

$$a_1 := \frac{k-1}{2} * \square_1 + a_n = 341.259\ \mathrm{m/s}. \tag{15}$$

The time dependence of the path of a macroparticle in a compression wave is given by equation (16) (Stanyukovich, 1971):

$$x(t) := x_0 - \left(\frac{k+1}{k-1} * a_1 * \left(\frac{t_0 - t_1}{t_0 - t}\right)^{\frac{k-1}{k+1}} - \frac{2}{k-1} * n\right) * (t_0 - t). \tag{16}$$

For a given value of the coordinate $x_2 = 1$ m, by the method of successive approximations from an arbitrarily given value *t*, for example, 0.01 s, the time t_2. To do this, there is the built-in Given-Find operator, writing it in this form (the logical equality sign "=" is entered by simultaneously pressing the "Ctrl" + "=") (17):

$$\text{Given: } t := 0.01\ x(t) = x_2;\ t_2 := \text{Find}(t) = 0.0082766. \tag{17}$$

Checking the time t_2 by calculating the coordinate x_2 according to the formula (16): $x(t_2) = 1.000000$, which coincides with the accepted value $x_2 = 1$. Calculation of the speeds of sound and particles of the flow at the second point (18, 19):

$$a_2 := a_1 * \left(\frac{t_0 - t_1}{t_0 - t_2}\right)^{\frac{k-1}{k+1}} = 354.305\ \mathrm{m/s}, \tag{18}$$

$$c_2 := \frac{2}{k-1} * (a_2 - a_n) = 71.526\ \mathrm{m/s}. \tag{19}$$

Using the dependence of pressure in a centred compression wave on coordinates and time (20):

$$p(x,t) := \left(\frac{2}{k+1}\right)^{\frac{2k}{k-1}} p_n * \left(1 + \frac{k-1}{2a_n} * \frac{x - x_0}{t - t_0}\right)^{\frac{2k}{k-1}}, \tag{20}$$

where the partial derivative was analytically found, $\mathrm{d}p(x,t)/\mathrm{d}x$ which in mathematics is usually denoted by $p'_x(x,t)$ (21) (Bronstein and Semendyaev, 1986):

$$p'_x(x,t) := k * \left(\frac{2}{k+1}\right)^{\frac{2k}{k-1}} * \frac{p_n}{a_n} * \left(1 + \frac{k-1}{2a_n} * \frac{x - x_0}{t - t_0}\right)^{\frac{k+1}{k-1}} * \frac{1}{t - t_0}. \tag{21}$$

Value output: $p'_x(x_1, t_1) = -1.343 * 10^4$; $p'_x(x_2, t_2) = -2.107 * 10^4$. Checking the found formula (21) using the standard Mathcad partial derivative operator $\frac{\partial}{\partial x} p(x,t)$ (22):

$$Z(x,t) := \frac{\partial}{\partial x} p(x,t);\ Z(x_1, t_1) = -1.343 * 10^4;\ Z(x_2, t_2) = -2.107 * 10^4. \tag{22}$$

Using expression (16) for the macroparticle trajectory, the authors of this study pass to one variable *t* in expression (21) (23):

$$p'_x(t) := k * \left(\frac{2}{k+1}\right)^{\frac{2k}{k-1}} * \frac{p_n}{a_n} * \left(\frac{k-1}{2} * \frac{a_1}{a_n}\right)^{\frac{k+1}{k-1}} * \frac{t_1 - t_0}{(t-t_0)^2}. \quad (23)$$

Check: $p'_x(t_1) = -1.343 * 10^4$ Pa/m; $p'_x(t_2) = -2.107 * 10^4$ Pa/m. The specific volume of gas is determined through pressure as a function of time (24):

$$p(t) := p_n * \left(\frac{a_1}{a_n}\right)^{\frac{2k}{k-1}} * \left(\frac{t_1 - t_0}{t - t_0}\right)^{\frac{2k}{k+1}}, \quad (24)$$

from the isentropic equation ($\upsilon/\upsilon_n = (p_n/p)^{1/k}$) (25):

$$\upsilon(t) := \frac{1}{k} * \left(\frac{a_1}{a_n}\right)^{\frac{-2}{k-1}} * \left(\frac{t_1 - t_0}{t - t_0}\right)^{\frac{-2}{k+1}}. \quad (25)$$

The derivative of the path *x*(*t*) (16) with respect to time gives the velocity (26):

$$?(t) := \frac{2}{k-1} * a_n * \left(\frac{a_1}{a_n} * \left(\frac{t_0 - t_1}{t_0 - t}\right)^{\frac{k-1}{k+1}} - 1\right). \quad (26)$$

Check: $c(t_1) = 6.296$ m/s; $c(t_2) = 71.526$ m/s. The velocity can also be determined through the derivative operator (27):

$$c(t) := \frac{d}{dt} x(t): c(t_1) = 6.296; c(t_2) = 71.526. \quad (27)$$

Then the path travelled by the particle in time d*t* (reference formula) (28):

$$dx = c(t) * dt = \frac{d}{dt} x(t) * dt. \quad (28)$$

Check: $\Delta x := \int_{t_1}^{t_2} c(t) * dt = 0.2$; $\Delta x := \int_{t_1}^{t_2} \frac{d}{dt} x(t) * dt = 0.2$; $x_2 - x_1 = 0.2$. Substituting the quantities (25), (23) and (28) into the expression for the specific work of moving an element of the medium as a whole – the convective component of the work of pressure forces in an unsteady flow (11), and finally obtain $w_{conv} = -\int_1^2 \upsilon * \frac{dp}{dx} dx$ (29):

$$w_{conv} := -\int_1^2 \upsilon(t) * p'_x(t) * \frac{d}{dt} x(t) * dt = 2538.181 \text{ J/kg}. \quad (29)$$

We determine the change in the specific kinetic energy of the flow element when it moves between points *1* and *2* (30):

$$\frac{c_2^2}{2} - \frac{c_1^2}{2} = 2538.181 \text{ J/kg}. \quad (30)$$

According to the energy equation for non-stationary flow (11), work (29) should be equal to the change in specific kinetic energy (30). Since this is equality (31):

$$\frac{c_2^2}{2} - \frac{c_1^2}{2} = w_{conv} = -\int_1^2 \upsilon * \frac{\partial p}{\partial x} dx = 2538.181 \text{ J/kg}, \quad (31)$$

is satisfied, then, therefore, both the energy equation (11) for a non-stationary flow and the calculation of the work w_{conv} are valid. Obviously, without the use of the Mathcad system, such a calculation would be very difficult and not visual.

Figure 3 shows path $x(t)$, particle velocity $c(t)$, sound speed $a(t)$, and pressure $p(t)$ versus time. Figure 4 shows a plot of the partial derivative of pressure with respect to the x-coordinate as a function of time. To build graphs, it is required to set the time range t:= 0.0001..0.083, where the first digit indicates the value of the first element, the value of the second element is indicated with a comma, and with a semicolon ";" the value of the last element. A semicolon when specifying a range is displayed as two periods (..). Figure 5 shows a three-dimensional pressure histogram given by the pressure matrix p (20) in the form (32):

$$p_{i,j} := \left(\frac{2}{k+1}\right)^{\frac{2k}{k-1}} p_n * \left(1 + \frac{k-1}{2a_n} * \frac{x_i - x_0}{t_i - t_0}\right)^{\frac{2k}{k-1}}, \tag{32}$$

where i:= 1..11; j:= 1..11.

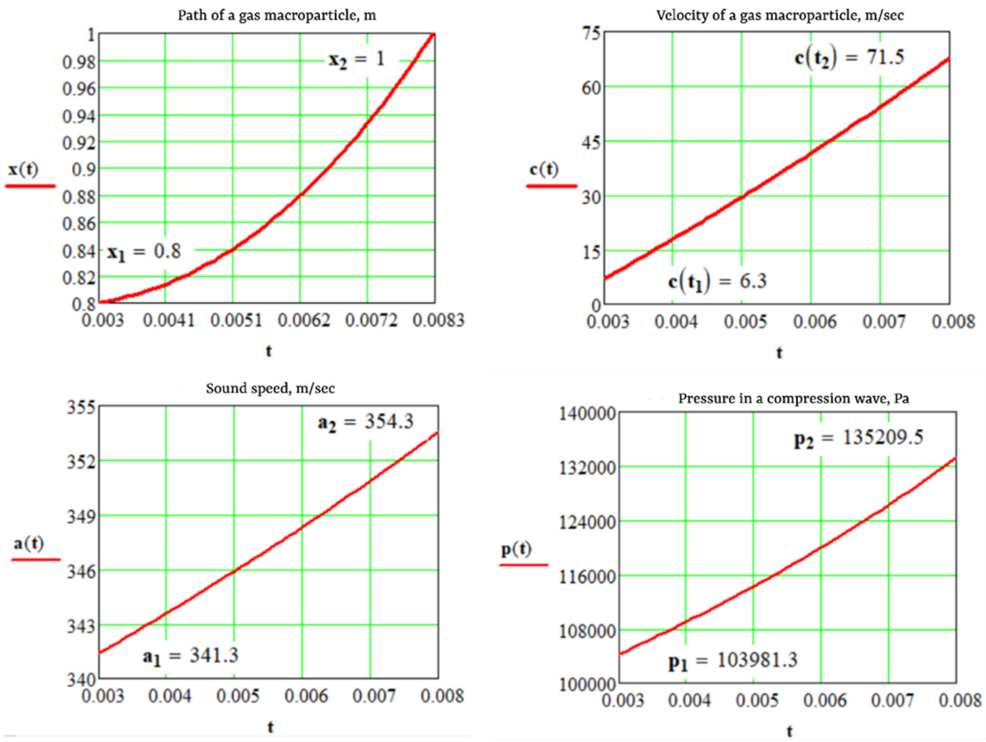


Figure 3 Plots of path, particle velocity, sound speed, and pressure versus time

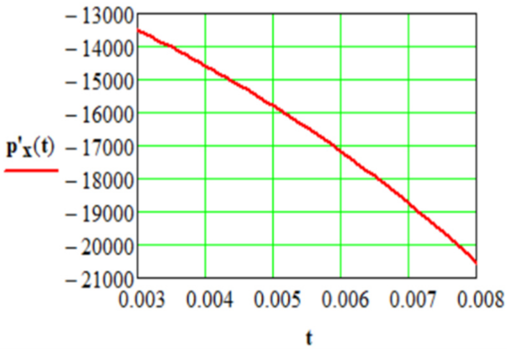


Figure 4. Partial derivative of pressure $p'_x(t)$, Pa/m along the x-coordinate as a function of t

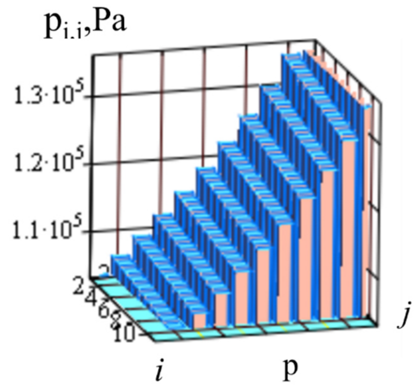


Figure 5. 3D pressure histogram

We proceed to the calculation of the local component of the work of the pressure forces w_{loc}, which is included in the energy equation for a non-stationary flow (12). The partial derivative of pressure (20) was calculated with respect to time in Mathcad (33):

$$p'_t(x,t) := \frac{\partial}{\partial t} p(x,t),\ p'_t(x_1,t_1) = 4.667 * 10^6 \text{ Pa/s}, \tag{33}$$

or analytically (34):

$$p'_x(x,t) := \frac{-2}{k+1} * \left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}} * \frac{p_n}{a_n} * \left(1 + \frac{k-1}{2a_n} * \frac{x-x_0}{t-t_0}\right)^{\frac{k+1}{k-1}} * \frac{x-x_0}{(t-t_0)^2}. \tag{34}$$

Checking the correctness of this formula: $p'_t(x_1,t_1) = 4.667 * 10^6$ Pa/s. We pass to one variable t (35):

$$p'_t(t) := \frac{35}{6} * p_n\left(\frac{6a_1}{5a_n} * \left(\frac{t_1-t_0}{t-t_0}\right)^{\frac{1}{6}} - 1\right) * \left(\frac{a_1}{a_n}\right)^6 * \frac{t_1-t_0}{(t-t_0)^2}. \tag{35}$$

$$p'_t(t) := \frac{35}{6} \cdot p_н \left(\frac{6a_1}{5a_н} \cdot \left(\frac{t_1 - t_0}{t - t_0} \right)^{\frac{1}{6}} - 1 \right) \cdot \left(\frac{a_1}{a_н} \right)^6 \cdot \frac{t_1 - t_0}{(t - t_0)^2}$$

Now there is the local component of the pressure change work (36):

$$w_{loc} := -\int_{t_1}^{t_2} \upsilon(t) * p'_t(t) * dt = -25224.01 \text{ J/kg}. \tag{36}$$

The equality between the found value of the work w_{loc} and the left side of the energy equation (12) (37):

$$\frac{a_1^2 - a_2^2}{k-1} + \frac{c_1^2 - c_2^2}{2} = -25224.01 \text{ J/kg}, \tag{37}$$

proves the validity of writing the FLT of a non-stationary flow in the form (12).

3.4. Method of experimental determination energy costs to overcome hydrodynamic resistance in unsteady flow

The above-discussed method for calculating the work w_{conv} was based on the known dependence (20) of pressure $p(x, t)$ in a compression wave, followed by transitioning to the partial derivative (21). In the general case, the dependence *p*(*x, t*) is not known. However, in each section of the pipe, it is possible to measure pressure as a function of time. As work is defined during the displacement of a particle along a trajectory, it is also necessary to know the time of arrival at a given section of a "tagged" particle, indicated by finely dispersed aluminium powder. To register the moment of particle arrival at a specific section, the flow visualization method can be used (Markov, 1973). The energy equation (6) for the case of one-dimensional non-stationary flow of viscous gas (assuming gdz and δw_{tech} are both zero) can be written in integral form as follows (38):

$$w_{fr} = -\int_1^2 \upsilon * \frac{\partial p}{\partial x} dx + \frac{c_1^2}{2} - \frac{c_2^2}{2}. \quad (38)$$

To determine empirically the integral in (38), the authors of this study divide the dimensional section of the pipe Δx = 0.2 m, for example, into four parts, equal to the length δx = 0.05 m, with coordinates x_1:=0.8 m; $x_{2'}$:= 0.85m; x_3:= 0.9 m; x_4:= 0.95 m and x_2:= 1 m. At points *1, 2', 3, 4* and *2*, the authors of this study record the pressure with sensors as a function of time. The time of arrival of a "tagged" particle, which can be indicated by finely dispersed aluminium powder, in a given section of the pipeline can be recorded by the flow photovisualization method, which was used by K.P. Stanyukovich (1971) in the study of the kinematic characteristics of unsteady flows. In the calculation of the authors of this study, this time was: t_1= 2.941$*10^{-3}$ s; $t_{2'}$= 5.423$*10^{-3}$ s; t_3= 6.618$*10^{-3}$ s; t_4=7.523$*10^{-3}$ s; t_2= 8.277$*10^{-3}$ s. Using the pressure recording curves in each measuring section for the corresponding time points, the authors of this study find the pressure values (in the example, according to the formula (20), Pa:

- $p_1(x_1, t_1) = 1.04*10^5$; $p_2(x_{2'}, t_{2'}) = 1.166*10^5$; $p_3(x_3, t_3)=1.238*10^5$;
- $p_4(x_4, t_4)= 1.298*10^5$; $p_2(x_2, t_2) = 1.352*10^5$.

Knowing only these data, it is possible to calculate the partial derivatives with respect to the *x*-coordinate at the starting point of each measured area as the ratio of the pressure drop in this area at the moment of arrival of the particle at the beginning of the area to the length of the area (Pa/m) (39, 40):

$$p'_x(x_1, t_1) := \frac{p(x_{2'}, t_1) - p(x_1, t_1)}{\delta x} = -1.339 * 10^4 \text{ Pa/m}, \quad (39)$$

$$p'_x(x_{2'}, t_{2'}) := \frac{p(x_3, t_{2'}) - p(x_{2'}, t_{2'})}{\delta x} = -1.63 * 10^4 \text{ Pa/m}. \quad (40)$$

Similarly, $p'_x(x_3, t_3) := -1.805 * 10^4$ Pa/m; $p'_x(x_4, t_4) := -1.958 * 10^4$ Pa/m; $p'_x(x_2, t_2) := -2.107 * 10^4$ Pa/m. The average value of the derivatives in the first section *1-2'*: $p'_{cp1} := 0.5 * (p'_x(x_1, t_1) + p'_x(x_{2'}, t_{2'})) = -1.485 * 10^4$ Pa/m. Similarly, the average values of partial derivatives for the remaining sections are found (Pa/m): $p'_{cp2} := -1.718 * 10^4$; $p'_{cp3} := -1.882 * 10^4$; $p'_{cp4} := -2.032 * 10^4$. Specific volumes at measured points are determined from the adiabatic equation $\upsilon_i := \left(\frac{p_1}{p_i}\right)^{\frac{1}{k}}$: υ_1= 0.8 m^3/kg; $\upsilon_{2'}$= 0.737 m^3/kg; υ_3= 0.706 m^3/kg; υ_4= 0.683 m^3/kg; υ_2= 0.663 m^3/kg. The average values of specific volumes on the corresponding segments of the pathm^3/kg) were found: υ_{1mid}:= 0.5(υ_1+$\upsilon_{2'}$) = 0.768; υ_{2mid}:= 0.5($\upsilon_{2'}$+υ_3) = 0.722; υ_{3mid}:= 0.5(υ_3+υ_4) = 0.695; υ_{4mid}:=0.5($\upsilon_{4'}$+υ_2) = 0.673. The specific work is determined by the formula (41, 42):

$$w_{12} := -(\upsilon_{1mid} * p'_{mid1} + \upsilon_{2mid} * p'_{mid2} + \upsilon_{3mid} * p'_{mid3} + \upsilon_{4mid} * p'_{mid4}) * \delta x, \quad (41)$$

$$w_{12} = 2527.507. \quad (42)$$

Relative error due to the small number of sections (four) (w_{conv} = 2538.181) $\frac{w_{conv} - w_{12}}{w_{conv}} =$

0.421%. Substituting the found work w_{12} instead of the integral in (38), the authors of this study find the work of friction in an unsteady flow. Thus, in order to calculate the work of friction using the energy equation for unsteady flow (38), it was necessary to measure the pressure in the dimensional sections in time, calculate the partial derivatives in the boundary sections using the proposed method, and then, using the formula (41), determine the work of the pressure forces in displacement environment element as a whole (w_{conv}).

4. Discussion

The quality of research carried out on approbation of the equation of the first law of thermodynamics for unsteady flow on a compression wave in the Mathcad system to detect errors and problems in the functioning of this system, and to increase the efficiency of the application of the Mathcad system, is one of the most urgent conditions of our time, and some problems require immediate solutions. Mathcad can be used to study the properties of compression waves and calculate the work of friction in arbitrary unsteady flows, Mathcad is a symbolic and numerical system that provides a convenient interface for mathematical modelling and analysis of various physical phenomena, using the appropriate equations of fluid mechanics and thermodynamics, it is possible to explore the properties of compression waves in non-stationary flows, Mathcad allows defining and solving differential equations that describe these processes, and visualizing the results in the form of graphs and tables. Calculation of the work of friction in any unsteady flow includes the analysis of hydrodynamic parameters such as pressure, velocity and viscosity, as well as taking into account the geometry and properties of the environment. Mathcad has tools for performing friction calculations and modelling, including the ability to create user-defined functions and perform numerical integration. Mathcad provides tools for performing calculations and friction simulations, including the ability to create user-defined functions and perform numerical integration, in general, Mathcad is a powerful tool for modelling and analysing a wide range of physical phenomena, such as the work of a pressure wave and friction in unsteady flows.

This study, conducted on the features of testing the equation of the first law of thermodynamics for an unsteady flow on a compression wave in the Mathcad system, made it possible to better understand the causes of errors during operation, especially during the development of this system, to assess the possibility of solving these problems and to identify at what stage they may appear. The Mathcad calculation program offers a wide range of functions for the analysis and visualization of various parameters of the compression wave, the software allows building graphs showing the time dependence of the paths, it also allows building graphs showing the dependence of the flow velocity and sound speed on time. The program developed in the Mathcad package can be used both to study the properties of compression waves and to calculate the work of friction in any unsteady flow, it allows building a trajectory, flow velocity, sound speed, pressure and other parameters associated with compression waves, also the program can be used to analyse and study the properties of compression waves in various conditions and environments, it helps to understand the behaviour of compression waves, their propagation, interaction with the environment and other factors.

It is worth noting that in the development of calculations and modelling, the Mathcad system and its developers have made a powerful step forward over the past few years. Non-stationary flows are characterized by a change in parameters over time and the presence of such dynamic phenomena as transients, transients and oscillations, the application of thermodynamics to non-stationary flows allows more accurately analysing and evaluating energy losses, system performance and optimizing its design, knowledge of energy equations for non-stationary flows allows taking into account the change in energy over time and taking into account dynamic conditions when designing efficient and energy-saving systems. Many modern technologies and engineering systems, such as power systems, heat exchangers, engines and turbomachines, operate in unsteady flow conditions, understanding the thermodynamic processes in such systems and developing appropriate energy equations for unsteady flows allows increasing and optimizing the efficiency of these technologies and create new innovative solutions, the use of energy equations for non-stationary flows improves the ability to predict and model the behaviour of the system in time, this allows predicting temperature, pressure, and other energy parameters at various points in the system depending on dynamic conditions and external factors, such forecasting and modelling is important for planning, control and optimization processes in various industries such as energy, manufacturing and transport.

According to the results of recent research by I.E. Olloberdi o'g'li (2023), unsteady flows are often observed in the inlet and outlet ducts of internal combustion engines and compressors, this is due to the dynamic processes that occur during the inlet and outlet of air and gases in internal combustion engines, such as gasoline engines with a spark ignition and self-ignition engines (diesel engines), the intake and exhaust gases are unstable processes, when the intake occurs, fuel is injected, the mixture of air and fuel is sucked into the cylinder and burned in the working cycle, after which exhaust gases are emitted. At the moment, it is necessary to improve the quality of unsteady flows that also occur in compressors used to compress gas or air, it is also necessary to raise the operation of compressors to a new level, it can be noted that the operation of this mechanism is based on the compression of gas or air, in which dynamic changes occur in the compressor path, air, or gas enters the compressor at a constant rate and is compressed by the compressor rotor. The whole mechanism of operation of these compressors and unsteady flow was analysed, as a result, it was decided that in both cases, unsteady flow in the inlet and outlet channels of engines and compressors can cause phenomena such as vortex formation, pressure drop, turbulence, and other dynamic effects, understanding and analysing these non-stationary processes are important for optimizing engine and compressor operation, increasing efficiency and reducing energy losses.

Referring to the definition of A. Gerőcs et al. (2023), the calculation and analysis of flows in the intake and exhaust pipelines of internal combustion engines is important for optimizing the gas exchange process. The processes of gas exchange associated with the entry of fresh air or gas into the cylinders and the removal of exhaust gases have a direct impact on the efficiency and performance of the engine. The intake process determines how fresh air or an air-fuel mixture enters the engine cylinders, this is an important stage on which the amount and quality of the charge supplied for fuel combustion depends on calculating the air flow in the intake piping, the shape, and dimensions of the intake manifold, pressure distribution and air flow can optimize cylinder filling and combustion efficiency. This indicates that there are coincidences with the work of this author, for example, that the exhaust process is important for the effective cleaning of exhaust gases in the cylinder after the end of the working cycle, also the correct configuration of the exhaust tract and flow design can help to effectively remove exhaust gases, reduce the amount of residual gases and improve the cleanliness of the cylinder before the next cycle. But, in this work, it was not taken into account the fact that an important property is that the patency of the intake and exhaust pipelines directly affects the efficiency and power of the engine, flow optimization improves filling the cylinders with fresh charge, reduces pressure and energy losses, reduces the volume of residual gases and increases the degree of removal of exhaust gases.

Researchers R. Gulaboski and V. Mirceski (2023) determined that complex mathematical programs and numerical methods are often used to calculate the parameters of unsteady flow. The calculation of non-stationary flows is a complex problem requiring the solution of the continuity equation, the equation of motion and the energy conservation equation for each moment of time. To model unsteady flows, numerical methods are commonly used, such as finite element, finite difference, and finite volume methods, these methods break the region of interest into smaller elements or cells and numerically solve the equations for each element or cell, taking into account boundary conditions and initial conditions. But, for a more correct solution of the equations of unsteady flow, it is required to use specialized software packages and development environments, such as ANSYS Fluent, COMSOL Multiphysics and Mathcad, these programs provide a wide range of tools for modelling and analysing unsteady flows, including the ability to specify geometry, boundary conditions and initial conditions, choice of numerical methods and visualization of results. Therefore, there are differences with this work that the author did not notice, namely the importance of such software tools that can be used to calculate not only simple flows, but also complex systems and devices, such as engines, compressors, turbines and blades, this allows analysing and optimizing flow characteristics, predict system behaviour over time, and studying in detail the parameters and effects associated with non-stationary flows.

M. Desch (2023) determined that the use of partial derivatives with respect to coordinates in thermodynamics makes it possible to consider specific work ($-\upsilon dp$) as the work of pressure forces on the movement of a medium element as a whole. In thermodynamics, specific work ($-\upsilon dp$) is the work done by pressure per unit mass of a substance in the process of its movement or deformation. Here, υ represents the specific volume of the element of the medium, and dp is the change in pressure. The results of this study of characteristics were analysed and more accurately considered, it can be supplemented with the fact that the specific work is related to mechanical energy and can be used to describe the displacement and deformation of the medium.

J. Lukasiak et al. (2023) showed by their work that the study of various aspects of unsteady flows and their behaviour using mathematical models and numerical methods is an important and productive work in the field of thermodynamics and hydrodynamics. Mathematical models make it possible to describe physical processes and phenomena associated with unsteady flows using equations and algorithms that can be solved numerically. These models allow creating virtual representations of the system, where it is possible to recreate and analyse various conditions, geometry, boundary conditions, and flow parameters. However, it was not indicated and considered in this work that at the moment numerical methods, such as the finite element method, the finite difference method and the finite volume method, are used to solve equations describing unsteady flows, these methods divide the area into a grid or elements, into which calculations and approximate numerical solutions are made. It can also be noted that this is due to the fact that the use of mathematical models and numerical methods allows to explore and analyse various aspects of non-stationary flows, such as dynamic effects, turbulence, vortex structures, changes in parameters over time, this helps to better understand the behaviour of flows, predict their characteristics and optimize systems and processes associated with non-stationary flows, because of this there is a difference between this work carried out and the work of the author.

As noted by G.P. Demelio et al. (2023), the use of the Mathcad system is a valuable tool for creating, solving and numerically analysing mathematical models, especially in the context of the study of unsteady flows. As part of the research, it is necessary to develop mathematical models that take into account various parameters, boundary conditions, and features of unsteady flows. These models must be implemented using the Mathcad system, which has powerful symbolic and numerical computing capabilities. It should also be noted that with the help of Mathcad and the developed mathematical models, it is possible to carry out numerical calculations in order to obtain numerical results and analyse the behaviour of unsteady flows. It is necessary to take the use of Mathcad in the study of unsteady flows to a new level, which will allow scientists or engineers to effectively create and analyse mathematical models, carry out numerical calculations and obtain results, this will contribute to a deeper understanding of unsteady flows, their characteristics and dynamic processes, which is important for the optimization and development of relevant systems and processes.

5. Conclusions

Therefore, in modern thermodynamics, equations of the FLT are provided for non-stationary (unsteady) processes occurring in open (flow) systems. However, energy equations for moving elements of non-stationary flow are not presented. Such flows occur in the intake and exhaust systems of internal combustion engines and compressors, where hydrodynamic losses are determined from the Bernoulli integral, written along the streamline for a specific moment in time. This calculation of work is conditional because the forces acting along the channel at a given moment and acting on the moving element of flow while it moving in time (along its trajectory) will be different. Since the energy equation (38) is obtained for a moving element of the medium moving along the trajectory for a certain period of time, it should be used for calculating friction work in non-steady flow. The FLT equations for non-stationary flow, containing partial derivatives with respect to coordinates and time, are not widely applied in practice due to the lack of a method for calculating these derivatives. The provided calculation program, implemented in the Mathcad software, allows for easy and visual determination of partial derivatives, corresponding integrals, and the construction of graphs depicting various quantities' dependencies on coordinates and time. This expands the application scope of thermodynamics.

In summary, the authors structured the following outcomes of this research. A thermodynamic (simplified) method for deriving the FLT equations for non-stationary flow is presented. These equations differ from analogous equations for steady flow due to the inclusion of terms containing partial derivatives of pressure with respect to coordinates or time. The derived equations are validated using an example of calculating the parameters of non-stationary flow in a centred compression wave. The FLT equations for non- stationary flow, containing partial derivatives with respect to coordinates and time, are not widely applied in practice due to the lack of a method for calculating these derivatives and integrals. A methodology for calculating the pressure force work $(-\upsilon(\partial p/\partial x)\mathrm{d}x)$ and $(-\upsilon(\partial p/\partial t)\mathrm{d}t)$ in a centred compression wave using the Mathcad software is outlined. The developed program for calculating gas parameters in a centred compression wave can be used to debug and visualize the calculation of individual blocks of complex mathematical models based on finite difference methods. A technique for the experimental determination of hydrodynamic energy losses in the intake and exhaust ducts of internal combustion engines using the equations obtained is given.

The existing programs for calculating the parameters of an unsteady flow, due to their complexity, cannot currently be used in thermodynamics courses. At the same time, the developed method for calculating the works in partial derivatives confirmed the validity of the energy equations for an unsteady flow, which makes it possible to introduce these equations into classical thermodynamics as widely as the energy equations for a flow system (control volume). This is the significance of this work.

References

1. Baehr, H. D., Stephan, K. 2011. Introduction. Technical applications. In: *Heat and Mass Transfer* (pp. 1-106). Berlin: Springer.
2. Borgnakke, C., Sonnag, R. E. 2009. *Fundamentals of thermodynamics, 10th edition*. New York: John Wiley and Sons.
3. Bronstein, I. N., Semendyaev, K. A. 1986. *Handbook of mathematics for engineers and university students*. Moscow: Nauka.
4. Callen, H. B. 1985. *Thermodynamics and introduction to thermostatics*. New York: John Wiley and Sons.
5. Demelio, G. P., Castellano, A., Bergamini, G. 2023. Design and implementation of an innovative wind power system for beach erosion protection and nourishment. *IOP Conference Series: Materials Science and Engineering*, 1275, 012032.
6. Desch, M. 2023. Design and certification for façade access systems: An alternative risk-based method. *Practice Periodical on Structural Design and Construction*, 28(4). https://doi.org/10.1061/PPSCFX.SCENG-1248
7. Gerőcs, A., Muncut, E., Komjaty, A., Lile, R., Bíró, I. 2023. Kinematic and dynamic study of an inertial system transforming rotational motion into unidirectional linear motion. *Tehnički Vjesnik*, 30(3), 980-987.
8. Gulaboski, R., Mirceski, V. 2023. Calculating of square-wave voltammograms – A practical on-line simulation platform. *Journal of Solid State Electrochemistry*. https://doi.org/10.1007/s10008-023-05520-y
9. Hołyst, R., Poniewierski, A. 2012. *Thermodynamics for chemists, physicists and engineers*. Dordrecht: Springer.
10. Kirillin, V. A., Sychev, V. V., Sheyndlin, A. E. 1983. *Technical thermodynamics*. Moscow: Energoatomizdat.
11. Kiryanov, D. V. 2006. *Self-tutor Mathcad 13*. St. Petersburg: BHV-Petersburg.
12. Kondepudi, D., Prigogine, I. 2015. *Modern thermodynamics: From heat engines to dissipative structurers*. New York: John Wiley and Sons.
13. Kraiko, A. N. 2007. *Theoretical gas dynamics*. Moscow: MIRT.

14. Krutov, V. I., Isayev, S. I., Kozhinov, A. I. 1991. *Technical Thermodynamics*. Moscow: Vysshaya Shkola.
15. Lavenda, B. H. 2010. *A new perspective on thermodynamics*. Berlin: Springer.
16. Łukasiak, J., Wiśnios, M., Rosiński, A. 2023. Method for evaluating the effectiveness of electrical circuit protection with miniature fuse-links. *Energies*, 16(2), 960.
17. Makarov, E. G. 2011. *Engineering calculations in Mathcad 15*. St. Petersburg: Piter.
18. Markov, S. B. 1973. Experimental study of velocity structure and hydraulic resistance in unsteady pressure turbulent flows. *News of the Academy of Sciences of the USSR*, 2, 65-74.
19. Morales-Rodriguez, R. 2016. *Thermodynamics: Fundamentals and its application in science*. London: InTechOpen.
20. Moran, M. J., Shapiro, H. N., Boettner, D. D., Balley, M. B. 2014. *Fundamentals of engineering thermodynamics*. New York: John Wiley and Sons.
21. Novikov, I. I. 1984. *Thermodynamics*. Moscow: Mashinostroenie.
22. Olloberdi o'g'li, I. E. 2023. Use of information technologies and computer mathematics systems in the process of teaching the subject of differential equations. *Eurasian Journal of Physics, Chemistry and Mathematics*, 17, 63-68.
23. Ryndin, V. V. 1987. On the issue of determining the energy costs to overcome hydrodynamic resistance in unsteady flow. *Proceedings of Universities. Engineering*, 1, 46-51.
24. Ryndin, V. V. 1991. Concept of work ($-vdp$) in thermodynamics. *Energy*, 10, 64-68.
25. Shushkevich, G. Ch., Shushkevich, S. V. 2010. *Computer technologies in mathematics. Mathcad 14*. Moscow: Grevtsov Publishing House.
26. Srivastava, R. C., Subit, K. S., Abhay, K. J. 2019. *Thermodynamics: A core course*. New Delhi: Prentice-Hall of India Pvt.Ltd.
27. Stanyukovich, K. P. 1971. *Unsteady motions of a continuous medium*. Moscow: Nauka.

Heat Transfer Research

Editors-in-Chief: **Zhixiong Guo , Oleg G. Penyazkov**
Regional Editor (India): **Partha S. Ghoshdastidar**
Regional Editor (China/Asia): **Ya-Ling He**
Regional Editor (Europe): **Bengt Sunden**
Regional Editor (USA): **Xinwei Wang**
Managing Editor: **Natalia K. Shveyeva**
Founding Editors: **James P. Hartnett , Thomas F. Irvine, Jr. , Oleg G. Martynenko**

Published **18** issues per year
ISSN Print: **1064-2285**
ISSN Online: **2162-6561**

Clarivate Analytics IF: **1.7**
Clarivate Analytics 5-Year IF: **1.4**
Clarivate Analytics Immediacy Index: **0.6**
Clarivate Analytics Eigenfactor: **0.00072**
Clarivate Analytics JCI: **0.43**
SJR: **0.318**
SNIP: **0.568**
CiteScore™:: **3.5**
H-Index: **28**

Indexed in
Clarivate ESCI
Clarivate SCIE
Scopus (Preview)
Engineering Village
EBSCO
British Library
Google Scholars (USA)
Ulrich's
CNKI (China)
Portico (Digital Preservation Agency) (USA)
Copyright Clearance Center (USA)
iThenticate (USA)
Scilit (Switzerland)

Validating the First Law of Thermodynamics for Unsteady Flow in a Compression Wave Using Mathcad

,

DOI: 10.1615/HeatTransRes.2023051072

Vladimir Ryndin

Amangeldy Karmanov

Akmaral Kinzhibekova

Rizagul Dyussova

Gulnara Abdullina