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Адрес редакции:

г. Алматы, ул. Сатпаева,
22, каб. 502, тел. 292-63-46
n. fedossenko @ ntu. kz

Капалова Н. А., Хомпыш А.

Разработка модификации алгоритма шифрования Эль-Гамаля на основе непозиционной системы счисления

Аннотация. Приведены основы использования непозиционных систем счисления в криптографии. Показан достоинства непозиционных систем счисления, высокая скорость вычисления по сравнению с позиционными системами счисления.

Разработана модификация алгоритма шифрования Эль-Гамаля на основе непозиционной системы счисления, которая в виде программного реализована.

Ключевые слова. Система остаточных классов, непозиционных систем счисления, алгоритма Эль-Гамаля, ключ, открытый текст, шифрование, дешифрование.

Kapalova N. A., Khompysh A.

A modification of the ElGamal encryption algorithm based on the non-position number system

Summary. This article presents the basics of using non-position number systems in cryptography. The advantages of non-position number systems, high computational speed in comparison with positional number systems are shown.

A modification of the ElGamal encryption algorithm based on the non-position number system has been developed, which is implemented as a software.

Key words: Residual class system, Non-position number systems, AlL-Gamal algorithm, key, plaintext, encryption, decryption.

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N. A. Ispulov*, T.G. Kissikov, A.Zh. Zhumabekov*, Zh.Dzh.Ospanova*, N.Zh. Zhuspekova***

(*Pavlodar State of University named after S. Toraighyrov, s. Pavlodar, nurlybek_79@mail.ru, almar89-89@mail.ru; ** The University of California, USA)

ABOUT PROPAGATION THE THERMOELASTIC WAVES IN THE ANIZOTROPIC MEDIUM OF A HEXAGONAL CRYSTAL SYSTEM

Summary. Actuality of study of wave processes laws in elastic mediums with thermomechanical effects is related to necessity to solve theoretical and applied problems of geophysics, seismology, mechanics of composites, and so on. Bound motion equations and heat conduction equations differ by difficulty and abundance of physical-mechanical parameters. Because of this part of deformable body mechanics - thermoelasticity - are being intensively developed. Within the bounds of this area, based on use of physical-mechanical properties of anisotropic mediums, bound heat and mechanical fields are being studied.

Keywords: Anisotropic medium, hexagonal crystal system, thermoelasticity, Fourier heat equation, harmonic waves, matricant.

1. Introduction

The theory of thermoelasticity deals with the study ofmutual interactions of thermal and mechanical fields in elastic bodies. It has vast applications in the various branches of Physics as well as in engineering, like materials engineering, mechanical engineering, nuclear engineering, and so forth. Theory of thermoelasticity is based on assumption that temperature distribution in an elastic object is governed by hyperbolic type parabolic-type partial differential equation as described by Fourier law of heat conduction [1–2]. According to Fourier law any thermal impulse is felt everywhere instantly in an object. Obviously it raised some serious concerns due to its unrealistic point of view. In order to circumvent this problem and to make it realistic a generalized theory of thermoelasticity was proposed which takes into account a finite thermal relaxation time. In this theory the temperature distribution is governed by hyperbolic type equations, which results in heat propagation in solids being considered as wave phenomenon instead of diffusion phenomenon.

The wave propagation in anisotropic inhomogeneous medium is considered. A new method of matricant has been developed. The method of matricant allows to investigate wave processing in anisotropic medium with various physical and mechanical properties [3].

The structure of matricant for the equation motion elastic medium equations, equations of thermo-mechanical medium has been established. Wave propagation in infinite and finite periodical inhomogeneous medium are studied.

In the paper [6], waves propagating along an arbitrary direction in a heat conducting orthotropic thermoelastic plate are presented by utilizing the normal mode expansion method in generalized theory of thermoelasticity with one thermal relaxation time. In the paper [7], authors studied the interaction of free harmonic waves with multilayered medium in generalized thermoelasticity by utilizing the combination of the linear transformation formation and transfer matrix method approach. Solutions obtained are generaland pertain to several special cases. Of these mention: (a) dispersion characteristics for a multilayered.

2. The matrix formulation of the propagation of thermoelastic waves

Propagation of thermoelastic waves in anisotropic medium described by the equations of motion to be solved together with the Fourier heat equation and the equation of heat flow, which have the form:

$$\frac{\partial\sigma_{XX}}{\partial X}+\frac{\partial\sigma_{XY}}{\partial Y}+\frac{\partial\sigma_{XZ}}{\partial Z}=\rho\frac{\partial^2 U_X}{\partial t^2}$$

$$\frac{\partial\sigma_{XX}}{\partial X}+\frac{\partial\sigma_{XY}}{\partial Y}+\frac{\partial\sigma_{XZ}}{\partial Z}=\rho\frac{\partial^2 U_X}{\partial t^2} \tag{1}$$

$$\frac{\partial\sigma_{XZ}}{\partial X}+\frac{\partial\sigma_{YZ}}{\partial Y}+\frac{\partial\sigma_{ZZ}}{\partial Z}=\rho\frac{\partial^2 U_Z}{\partial t^2}$$

$$\lambda_{ij}\frac{\partial\theta}{\partial x_j}=-q_i \tag{2}$$

$$\frac{\partial q_i}{\partial x_i}=-i\omega\beta_{ij}\varepsilon_{ij}-i\omega\frac{c_\varepsilon}{T_0}\theta \tag{3}$$

where σ_{ij} - stress tensor, ρ - density of the medium, λ_{ij} - thermal conductivity tensor, q_i - the vector of heat, ω - the angular frequency, β_{ij} - thermomechanical constants, $\beta_{ij}=\beta_{ji}$, ε_{ij} - the strain tensor, c_ε - specific heat at constant strain, $\theta=T-T_0$ - temperature increase compared with the temperature of the natural state T_0, $\left|\frac{\theta}{T_0}\right| \prec\prec 1$ for small deformations.

Physical and mechanical quantities are related by relation of Duhamel-Neumann:

$$\sigma_{ij}=c_{ijkl}\varepsilon_{kl}-\beta_{ij}\theta \tag{4}$$

Here c_{ij} - the elastic parameters, $c_{ijkl}=c_{jikl}=c_{ijlk}=c_{klij}$; ε_{kl} - the tensor Cauchy for small deformations.

For crystals of hexagonal system as coordinate three orthogonal axes of symmetry or inversion axes of the second order get out.

For a hexagonal class of crystals the relation of Duhamel - Neumann looks like:

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}(c_{11}-c_{12}) \end{pmatrix} \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{pmatrix} - \begin{pmatrix} \beta_{11} & 0 & 0 \\ 0 & \beta_{11} & 0 \\ 0 & 0 & \beta_{33} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \theta \qquad (4)^{/}$$

Equations (1), (2), (3), (4) and $(4)^{/}$ determine the relationship of mechanical stress and temperature as a function of the independent variables - the thermal field and deformation.

Thus, the relation (1) - $(4)^{/}$ constitute a closed system of thermoelasticity equations, which describes the propagation of thermoelastic waves.

Based on the method of separation of variables in the case of a harmonic function of time:

$$\left[U_i(x,y,z,t);\ \ \sigma_{ij}(x,y,z,t);\theta;\,q_z \right] = \left[U_i(z), \sigma_{ij}(z),\,\theta;\,q_z \right] e^{i(\omega t - mx - ny)} \qquad (5)$$

The system of equations (1) - (4) reduces to a system of differential equations of first order with variable coefficients which describes the propagation of harmonic waves:

$$\frac{d\vec{W}}{dz} = B\vec{W} \qquad (6)$$

Here $B = B\left[c_{ijkl}(z), \beta_{ij}(z), \omega, m, n\right]$- coefficient matrix whose elements contain the parameters of the medium in which waves propagate thermoelastic; m, n-components of the wave vector $\tilde{\kappa}$.

The vector $\vec{W}$ has the form:

$$\vec{W}(x,y,z,t) = \left[u_z(z), \sigma_{zz}, u_x(z), \sigma_{xz}, u_y(z), \sigma_{yz}, \theta, q_z\right]^t \exp(i\omega t - imx - iny) \qquad (7)$$

The symbol t indicates the transpose of the vector - a vector of strings - Column.

The system of differential equations (6) for non-isotropic medium of a hexagonal singony looks like:

$$\frac{dU_z}{dZ} = \frac{1}{c_{33}}\sigma_{zz} + \frac{c_{13}}{c_{33}} imU_x + \frac{c_{13}}{c_{33}} inU_y + \frac{\beta_{33}}{c_{33}}\theta$$

$$\frac{d\sigma_{zz}}{dZ} = -\rho\omega^2 U_z + im\sigma_{xz} + in\sigma_{yz}$$

$$\frac{dU_x}{dZ} = \frac{1}{c_{44}}\sigma_{zx} + imU_z \qquad \frac{d\sigma_{xz}}{dZ} = im\,\frac{c_{13}}{c_{33}}\sigma_{zz} + \left[-\rho\omega^2 + m^2\left(c_{11} - \frac{c_{13}^2}{c_{33}}\right) + \frac{c_{11}-c_{12}}{2}n^2\right]U_x + m\,n\left(c_{12} - \frac{c_{13}^2}{c_{33}} + \frac{c_{11}-c_{12}}{2}\right)U_y + $$

$$+\left(\frac{c_{13}}{c_{33}}\beta_{33} - \beta_{11}\right) im\,\theta$$

$$\frac{dU_y}{dZ} = \frac{1}{c_{44}}\sigma_{yz} + imU_z$$

$$\frac{d\sigma_{yz}}{dZ} = im\ \frac{c_{13}}{c_{33}}\sigma_{zz} + m\,n\left[c_{12} + \frac{c_{11}-c_{12}}{2} + c_{13}\right]U_x + \left(-\rho\omega^2 + m^2 c_{11} - n^2\frac{c_{13}^2}{c_{33}} + \frac{c_{11}-c_{12}}{2}m^2\right)U_y +$$

$$+\frac{c_{13}}{c_{33}}\beta_{33}in\,\theta$$

$$\frac{d\theta}{dZ} = -\frac{1}{\lambda_{33}}q_z$$

$$\frac{dq_z}{dZ} = -i\omega\frac{\beta_{33}}{c_{33}}\sigma_{zz} + \omega m\left(\frac{c_{13}}{c_{33}}\beta_{33} - \beta_{11}\right)U_x + \omega n\frac{c_{13}}{c_{33}}\beta_{33}U_y - i\omega\left(c_\varepsilon + \frac{\beta_{11}^2}{c_{11}}\right)\theta$$

The heterogeneity of the medium is assumed along Z. In constructing the coefficient matrix B is used as a representation of the solution (5), the system of equations (1) - (4) are in the derivatives along the coordinate Z and the excluded components of the stress tensor is not included in the boundary conditions. The multiplier $\exp(i\omega t - imx - iny)$ is omitted throughout.

In the structure of the matrix and vector - column boundary conditions in the bulk case for the hexagonal crystal system in the case of the symmetry axis of the second order and heterogeneity along the Z axis are given by:

$$B = \begin{bmatrix} 0 & b_{12} & b_{13} & 0 & b_{15} & 0 & b_{17} & 0 \\ b_{21} & 0 & 0 & b_{24} & 0 & b_{26} & 0 & 0 \\ b_{24} & 0 & 0 & b_{34} & 0 & 0 & 0 & 0 \\ 0 & b_{13} & b_{43} & 0 & b_{45} & 0 & b_{47} & 0 \\ b_{26} & 0 & 0 & 0 & 0 & b_{56} & 0 & 0 \\ 0 & b_{15} & b_{45} & 0 & b_{65} & 0 & b_{67} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{78} \\ 0 & -i\omega b_{17} & -i\omega b_{47} & 0 & -i\omega b_{67} & 0 & b_{87} & 0 \end{bmatrix}; \quad \vec{W} = \begin{pmatrix} u_z \\ \sigma_{zz} \\ u_x \\ \sigma_{xz} \\ u_y \\ \sigma_{yz} \\ \theta \\ q_z \end{pmatrix} \qquad (8)$$

From the structure of the coefficient matrix (8) that in the spatial case, the elastic waves of different polarization and the heat wave are interrelated.

The nonzero elements of the matrix of coefficients B b_{13} , b_{24} determine the mutual transformation of longitudinal and transverse X - polarized waves. Elements of b_{15}, b_{26} describe the relationship of transverse Y-polarization with the longitudinal wave. Nonzero element b_{45} defines the mutual transformation between the waves of transverse polarization.

The fact that the coefficient b_{17}:

$$b_{17} = \frac{\beta_{33}}{c_{33}}$$

means that the longitudinal wave is propagated from the thermoelastic effect.

Non-zero elements b_{47} and b_{67}:

$$b_{47} = \left(\frac{c_{13}}{c_{33}}\beta_{33} - \beta_{11}\right)im\ ; \quad b_{67} = \frac{c_{13}}{c_{33}}\ \beta_{33}in$$

indicate the effect on the elastic wave transverse polarizations thermoelastic effect. At the same time describes the effect b_{47} thermoelastic effect on the elastic shear wave of the X-polarization, and b_{67} effects thermoelastic effect on the transverse wave Y-polarization.

Similarly, for the thermo-elastic waves propagating in an anisotropic medium of cubic symmetry the coefficient matrix is constructed in the bulk case and the analysis of matrix coefficients. We also obtain the structure of the matrix of coefficients in the propagation of thermoelastic waves in anisotropic medium of hexagonal crystal systems in the plane XZ and YZ, defines the types of waves and the mutual transformation of waves of different polarizations.

3. Conclusion

Differential equations system of the first order with variable coefficients that are made by means of variable separation method are made (solution is presented as a plane harmonic wave). Coefficients matrix of anisotropic mediums of a hexagonal singony for three-, two-, and one-dimensional cases were obtained.

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Испулов Н.А., Кисиков Т.Ғ., Жұмабеков А.Ж., Оспанова Ж.Ж., Жуспекова Н.Ж.

Гексагоналды сингониялы анизотропты ортада термосерпімді толқындардың таралуы туралы

Түйіндеме. Термомеханикалық эффектімен болатын серпімді орталарда толқындық процестердің заңдылықтарды зерттеу актуалдығы, геофизика, сейсмология, композиттік материалдардың механикасының теориялық және қолданбалы есептерді шешуінде қажеттілігімен байланысты. Байланысқан қозғалыс теңдеулері мен жылуөткізгіштік теңдеулері физика–механикалық параметрлердің күрделігі мен көп болуымен ерекшеленеді. Осыған байланысты деформацияланатын қатты дене механикасының – термосерпімділік деген тарауы қарқынды дамып келеді. Осы бағыттың аясында анизотропты орталардың кейбір физика–механикалық қасиеттерін қолдана отырып, байланысқан жылулық және механикалық өрістер зерттеледі.

Кілт сөздері: Анизотропты орта, гексагоналды сингония, термоэластика, термосерпімді толқындар, Фурьенің жылулық теңдеуі, матрицант.

Испулов Н.А., Кисиков Т.Ғ., Жұмабеков А.Ж., Оспанова Ж.Ж., Жуспекова Н.Ж.

О распространении термоупругих волн в анизотропной среде гексагональной сингонии

Резюме. Актуальность исследования закономерностей волновых процессов в упругих средах с термомеханическим эффектом связана с необходимостью решения теоретических и прикладных задач геофизики, сейсмологии, механики композитных материалов и т.д. Связанные уравнения движения и уравнения теплопроводности отличаются сложностью и обилием физико-механических параметров. В связи с этим интенсивно развивается раздел механики деформируемого твердого тела, - термоупругость. В рамках этого направления, опираясь на использование определенных физико-механических свойств анизотропных средах, изучаются связанные тепловые и механические поля.

Ключевые слова: Анизотропная среда, гексагональная сингония, термоэластика, термоупругие волны, тепловое уравнение Фурье, матриցанты.

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